

Probing Ion Distributions in Charged Polymers Using Neutron, Light, and X-Ray Scattering

Carlos G. Lopez

cvg5719@psu.edu

<https://polyelectrolyte.science>

Materials Science and Engineering, Penn State University, University Park, PA
16802 USA

[Paper draft](#)



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and Mineral Sciences

Presentation Outline

- Counterion condensation: the central problem in polyelectrolyte physics
- Light scattering as a tool to study counterion condensation
- Beyond the two-phase model: insight from SANS and SAXS into counterion distributions



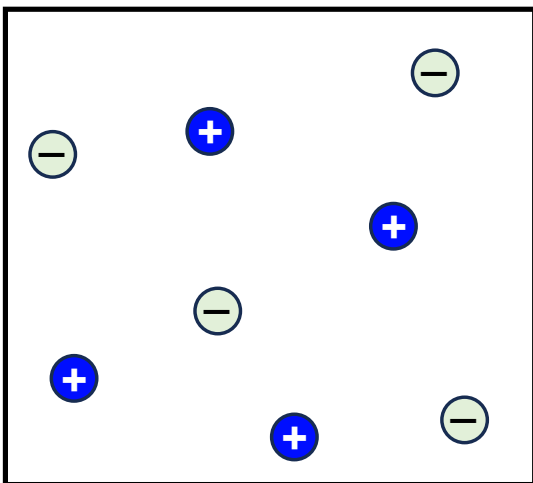
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Solutions of simple salts

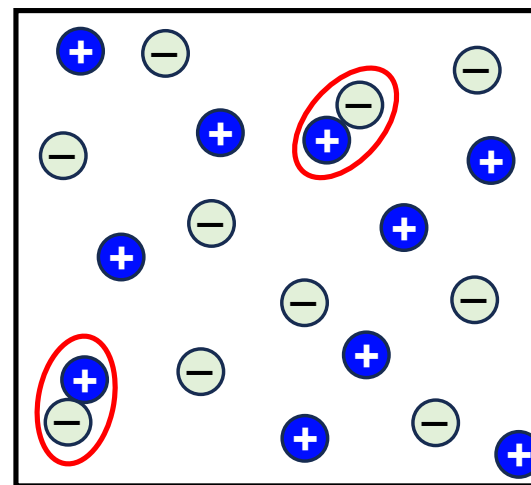
dilute



Individual ions

In distribution is described by Debye-Hückle theory (simple physics)

concentrated



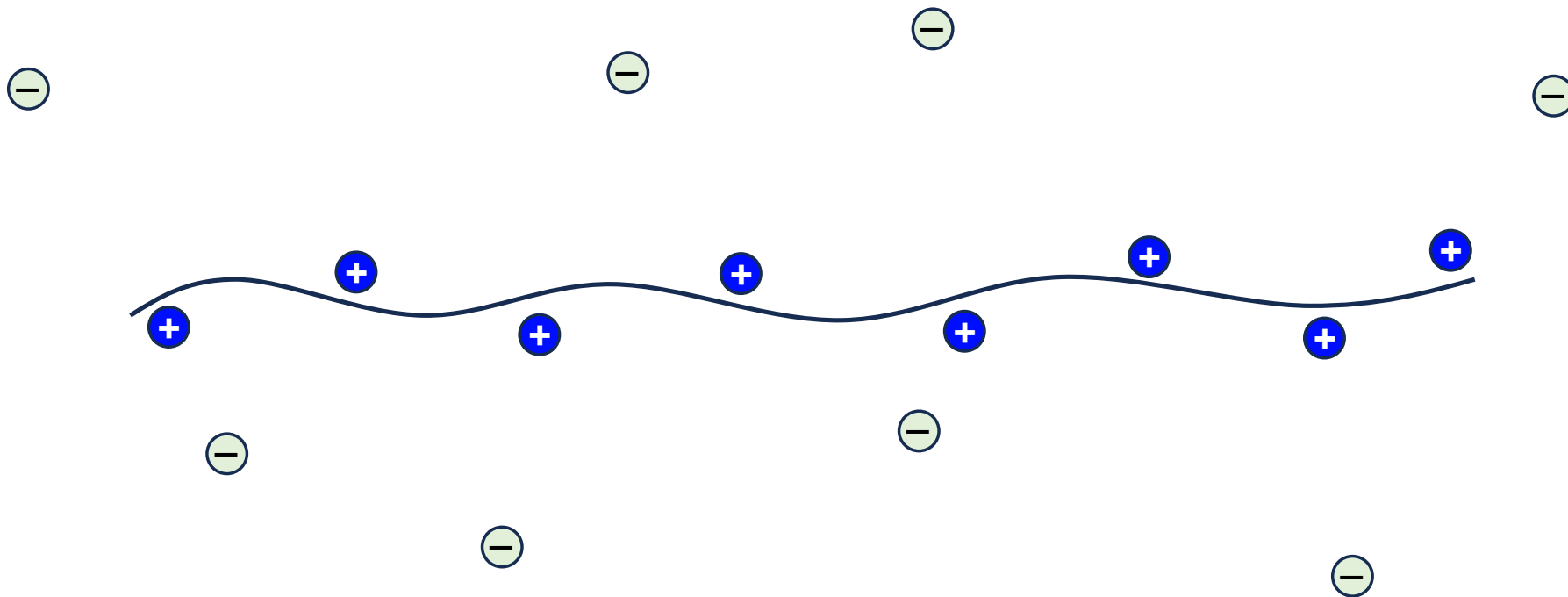
Ion-pair formation

Ion-pairing is set by solvation shells and follows a mass-action law (chemistry becomes important)



Polyelectrolyte solutions

Polyelectrolytes are polymers with ionic groups along their backbone

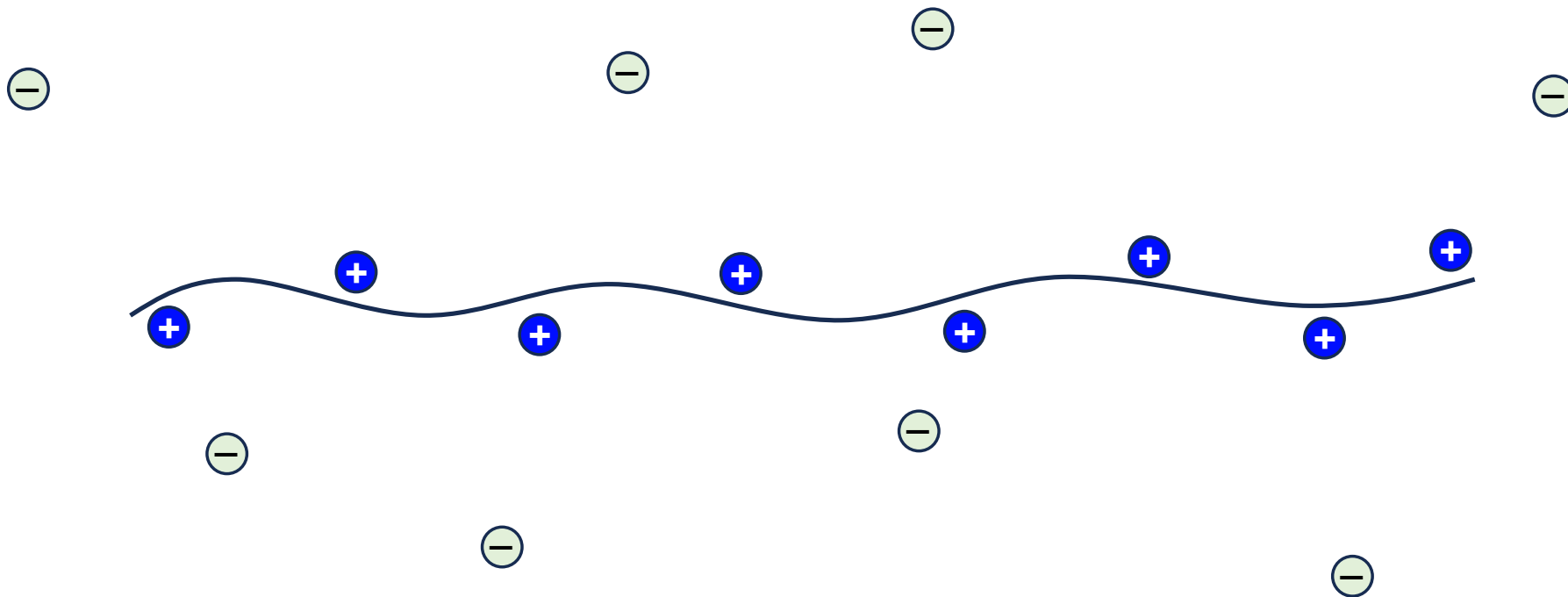


Side groups are chemically fixed onto the backbone → no mobility



Polyelectrolyte solutions

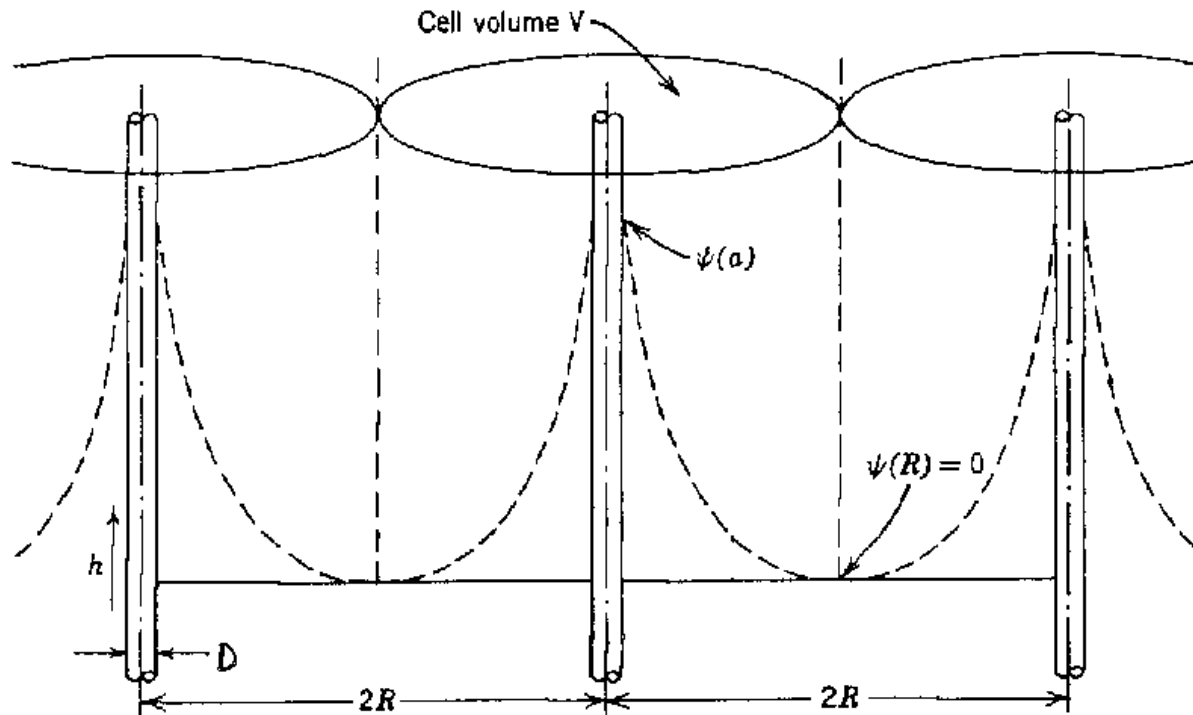
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Already in the 1940s experimentalists noticed that there were 'missing counterions'

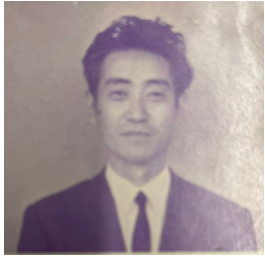


Katchalsky's cell model



Aharon Katchalsky
(1914-1972)

Counterions accumulate near the polyion and become
osmotically inactive



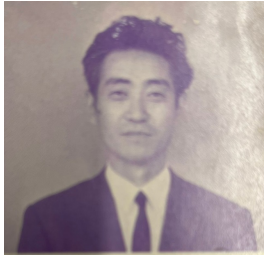
Fumio Oosawa
(1922-2019)

Polyelectrolytes: Oosawa-Manning theory of condensation



Gerald Manning
(b.19??)



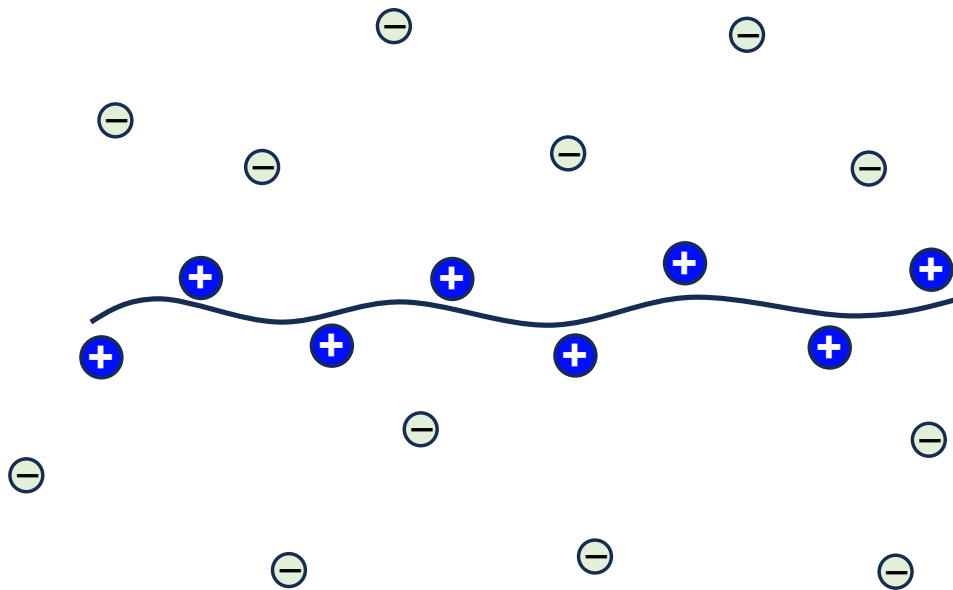


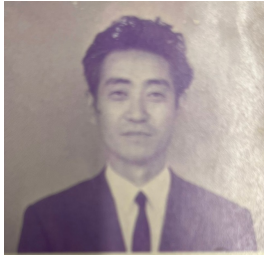
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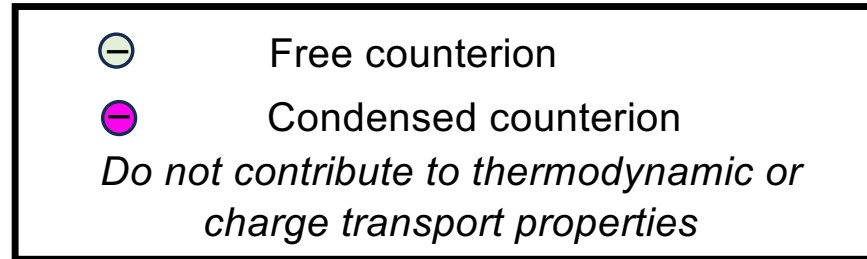
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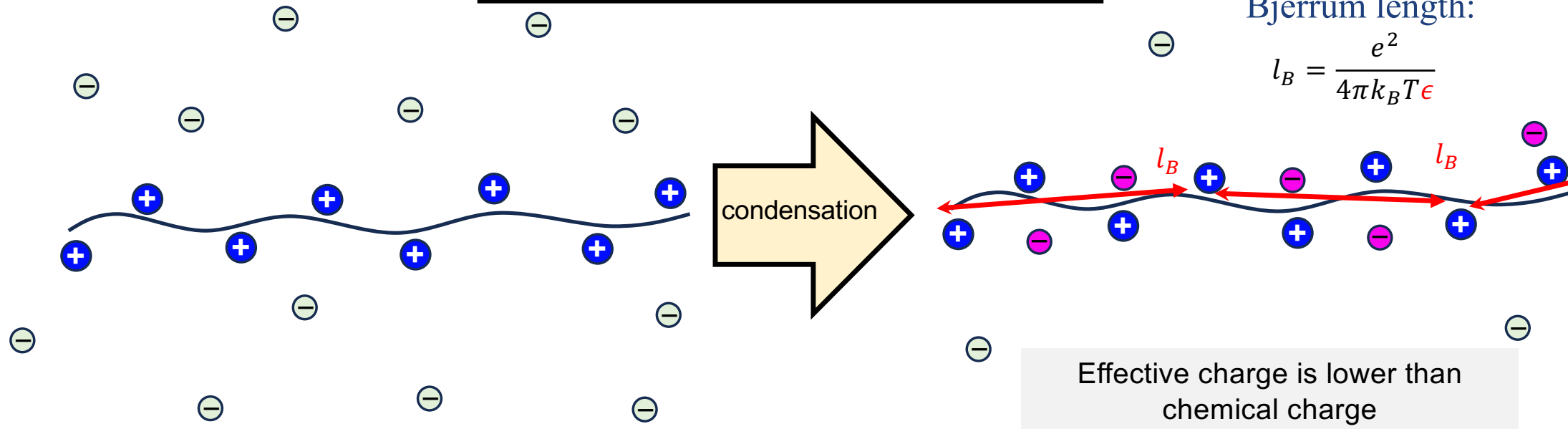
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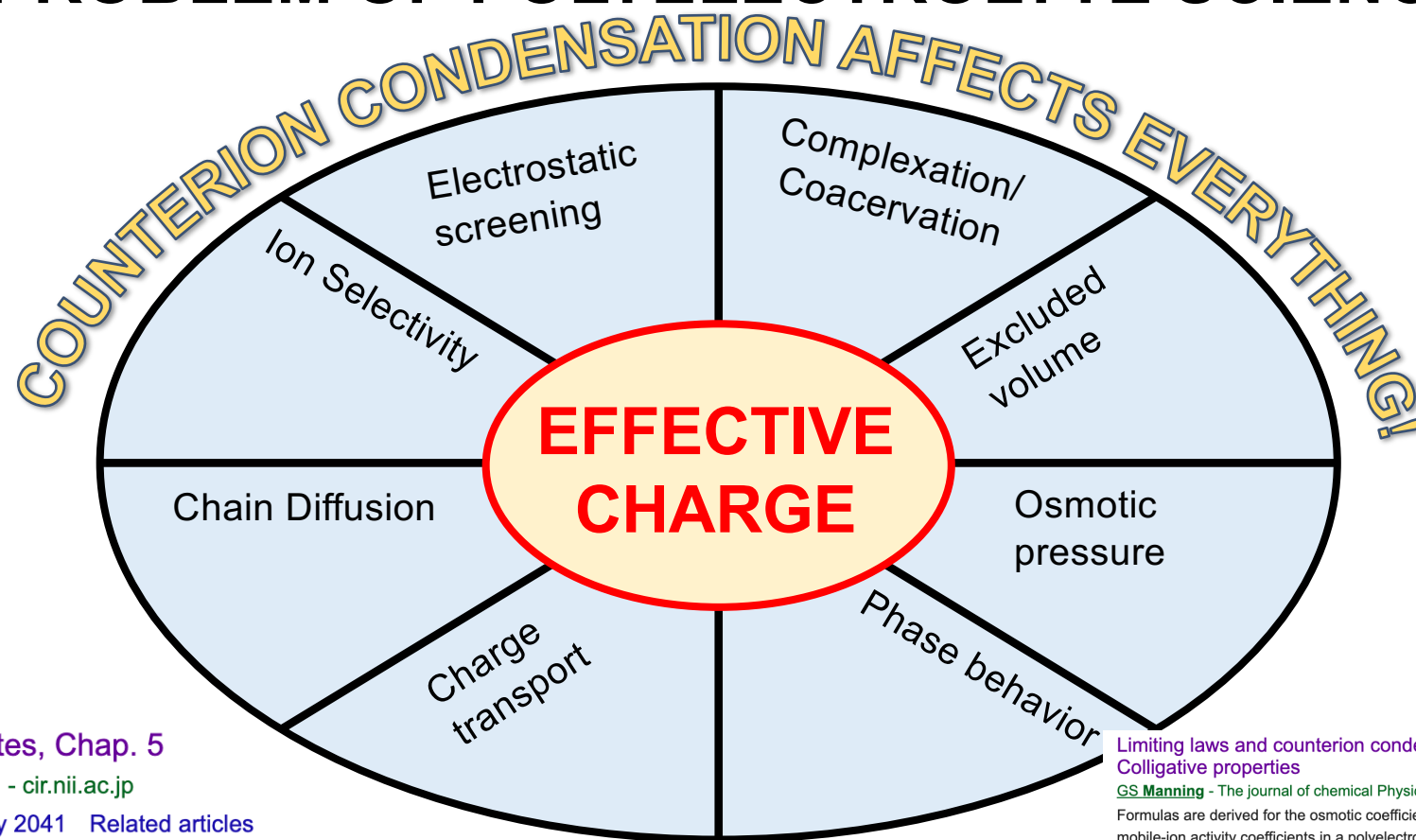
Gerald Manning
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Bjerrum length:

$$l_B = \frac{e^2}{4\pi k_B T \epsilon}$$



COUNTERION CONDENSATION IS THE CENTRAL PROBLEM OF POLYELECTROLYTE SCIENCE



[CITATION] Polyelectrolytes, Chap. 5

F Oosawa - (No Title), 1971 - cir.nii.ac.jp

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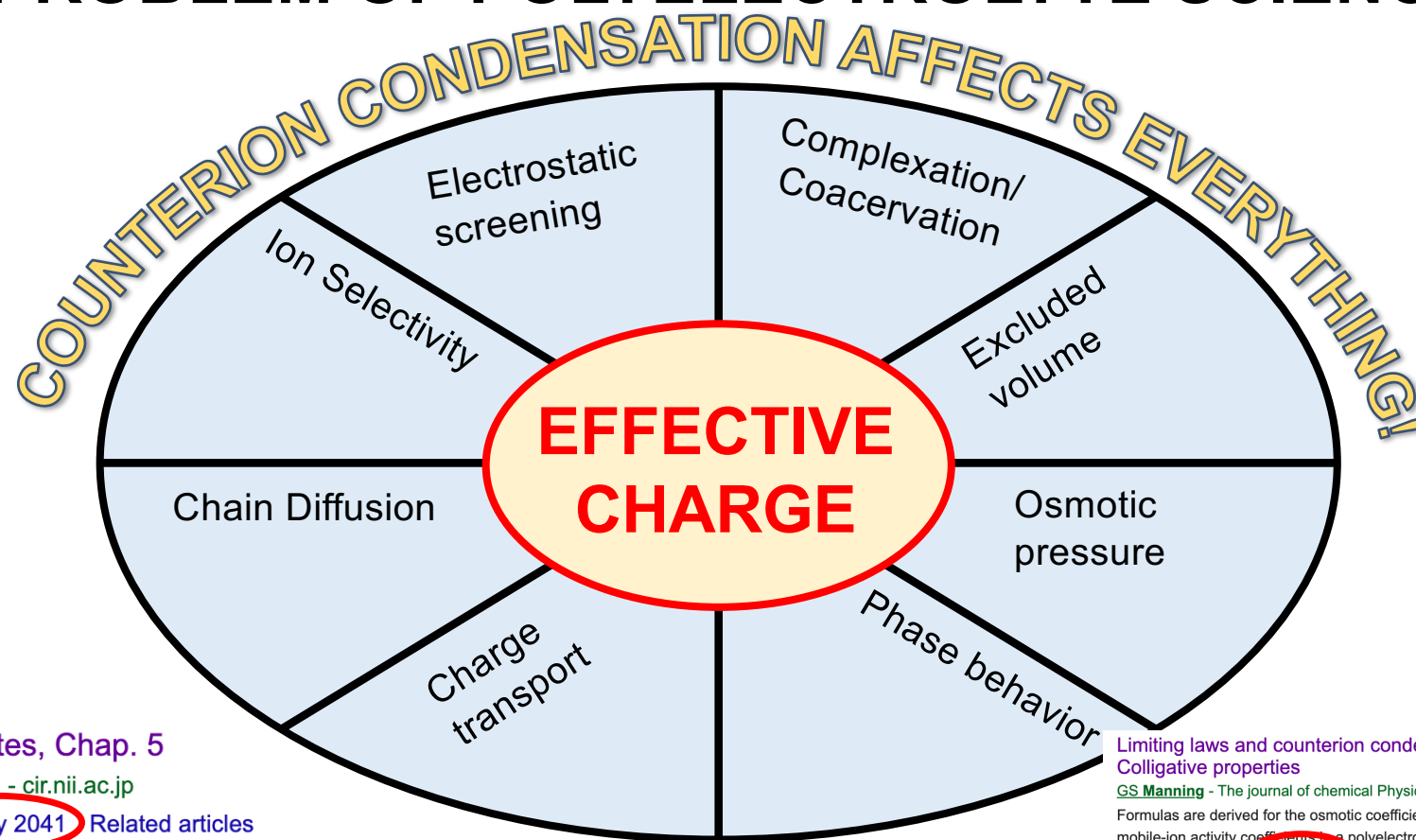
Formulas are derived for the osmotic coefficient, the Donnan salt-exclusion factor, and the mobile-ion activity coefficients in a polyelectrolyte solution with or without added sample salt. ...

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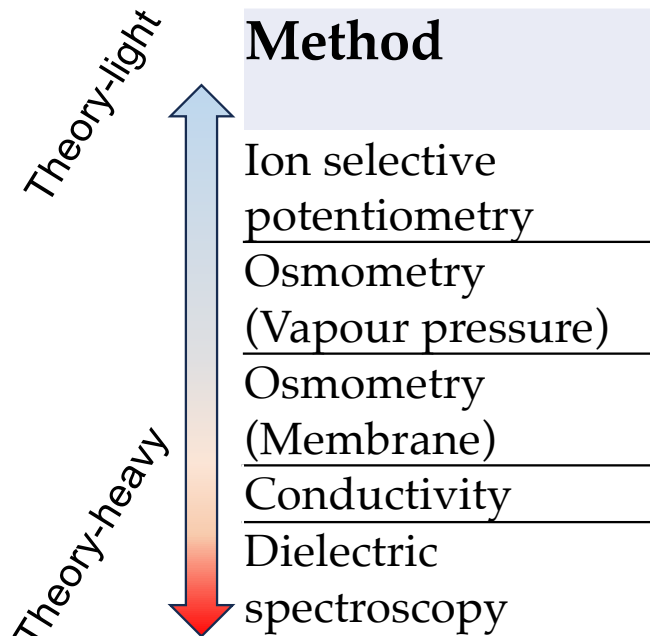
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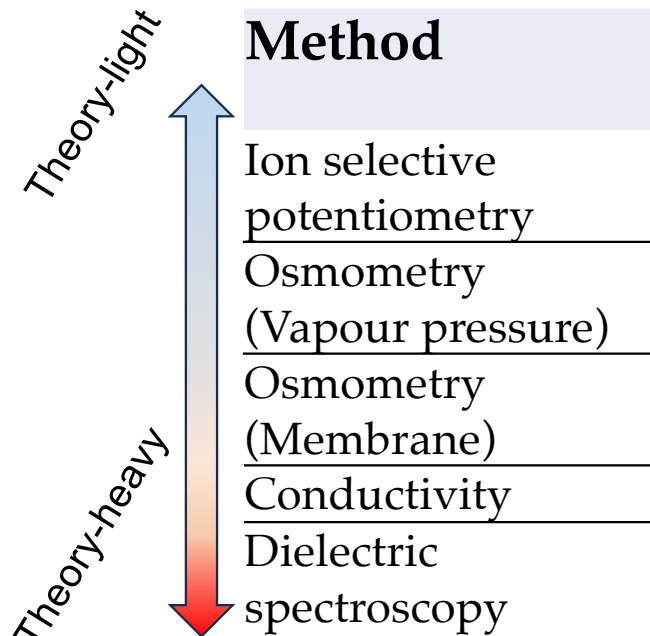
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METHODS FOR DETERMINING COUNTERION CONDENSATION

- Various methods yield different results for the fraction of free counterions.



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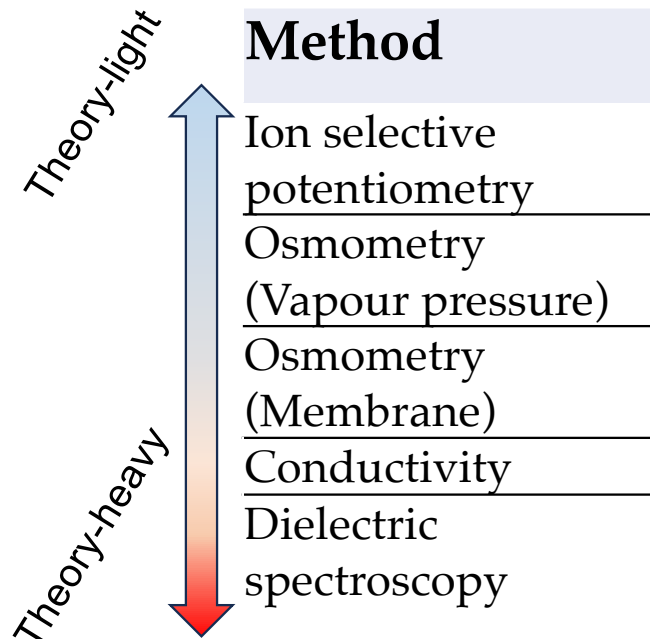
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$$\Pi = k_B T f c$$

Each free counterion contributes $\approx k_B T$ to the osmotic pressure.



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DRAWBACK: very difficult for non-aqueous systems.



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LIGHT SCATTERING FROM SALT-FREE POLYELECTROLYTES

Static structure factor:

$$S(\mathbf{q}) = \frac{1}{N} \sum_{j=1}^N \sum_{k=1}^N e^{-i\mathbf{q} \cdot (\mathbf{R}_j - \mathbf{R}_k)}$$



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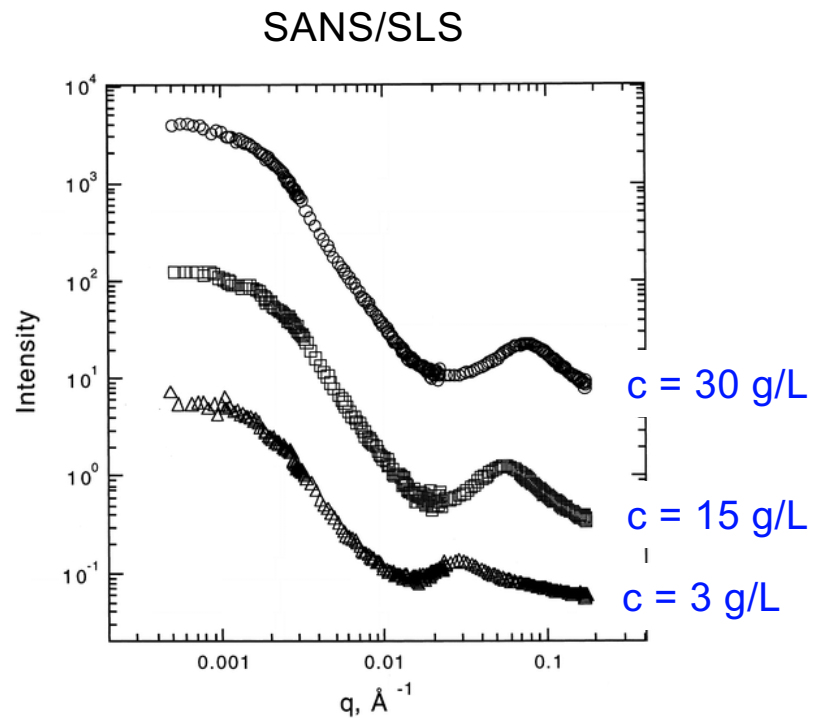
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Polyelectrolytes should scatter weakly at low q !

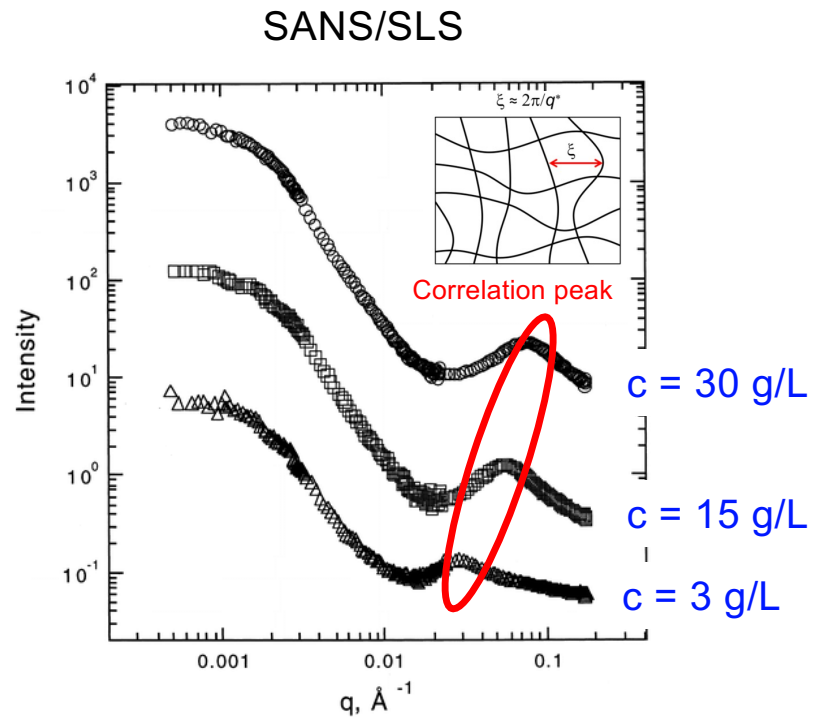
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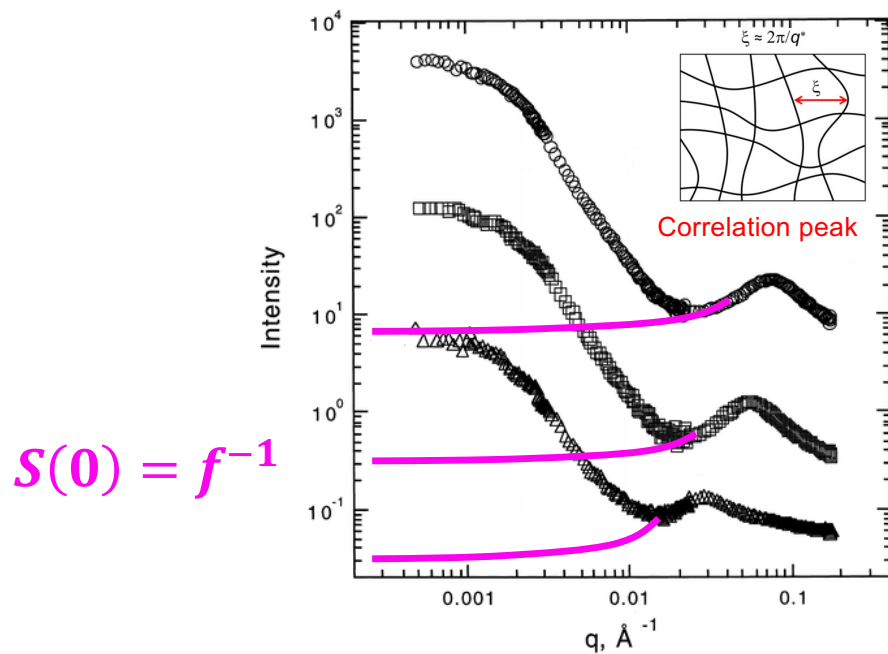


SCATTERING FROM SALT-FREE POLYELECTROLYTES



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SANS/SLS



Theory expects extremely weak scattering by polyelectrolytes at low q but experiments show a huge low- q upturn

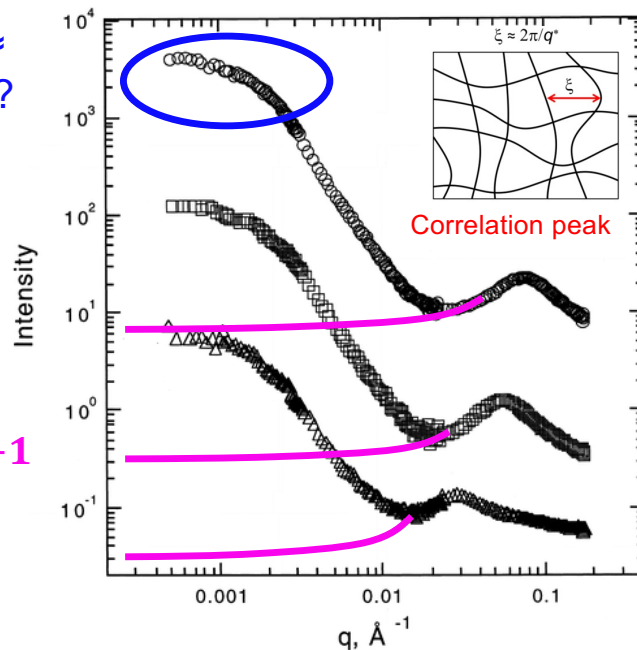


SCATTERING FROM SALT-FREE POLYELECTROLYTES

SANS/SLS

Low q upturn $\approx$
large domains?

$$S(0) = f^{-1}$$



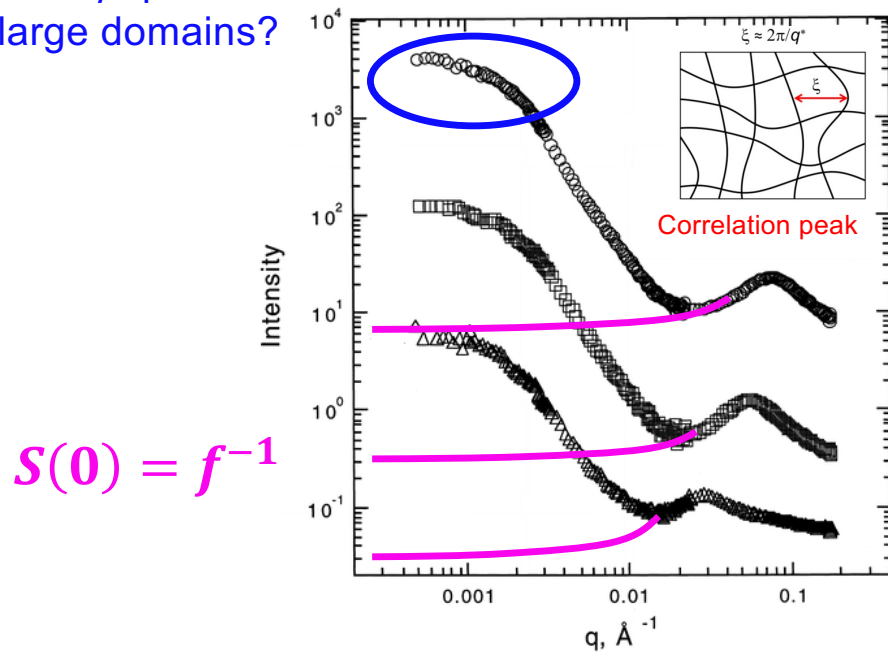
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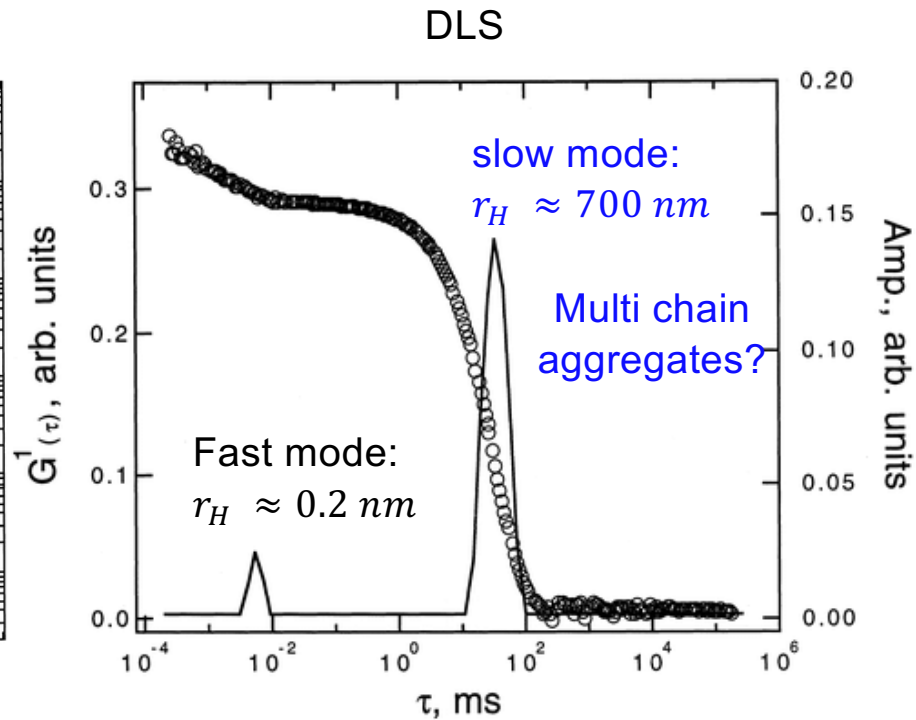
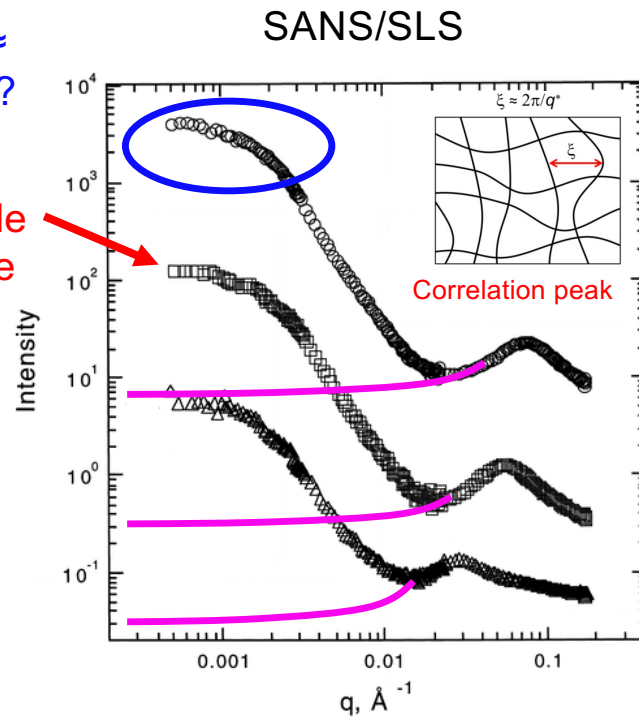
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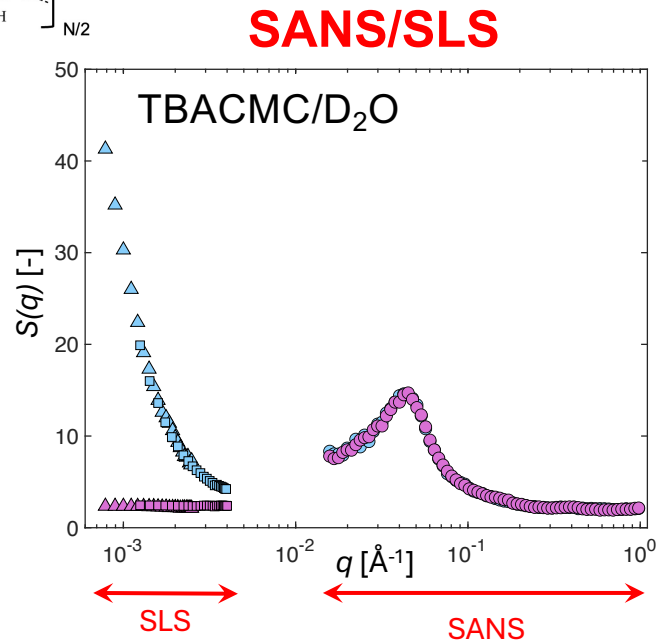
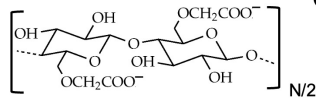
This does not
give a reasonable
osmotic pressure



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FILTRATION (MOSTLY) REMOVES LOW-Q UPTURN



Filter

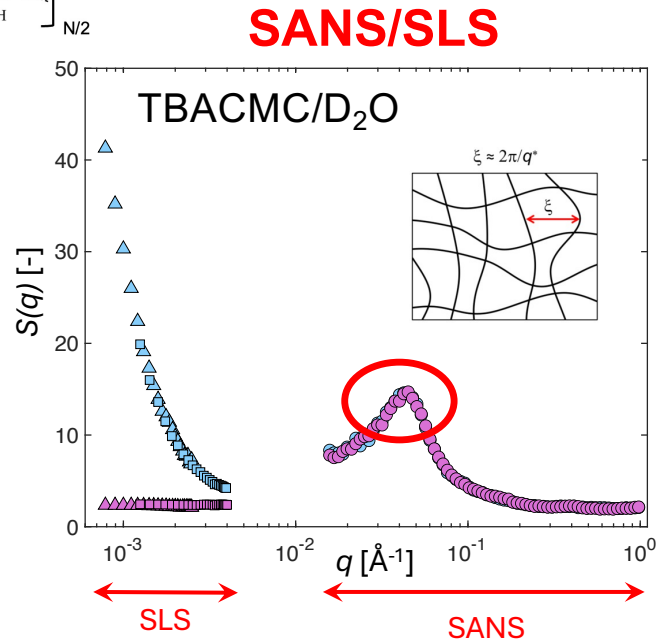
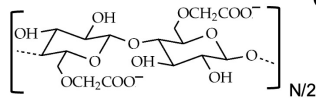
0.1 μm pore size

0.8 μm pore size

- Filtration through sufficiently small pores suppresses upturn & slow mode.
- Mesh size is not affected by filtration → no polymer removed (density measurements further support this)
- Solutions are stable for at least 1 month



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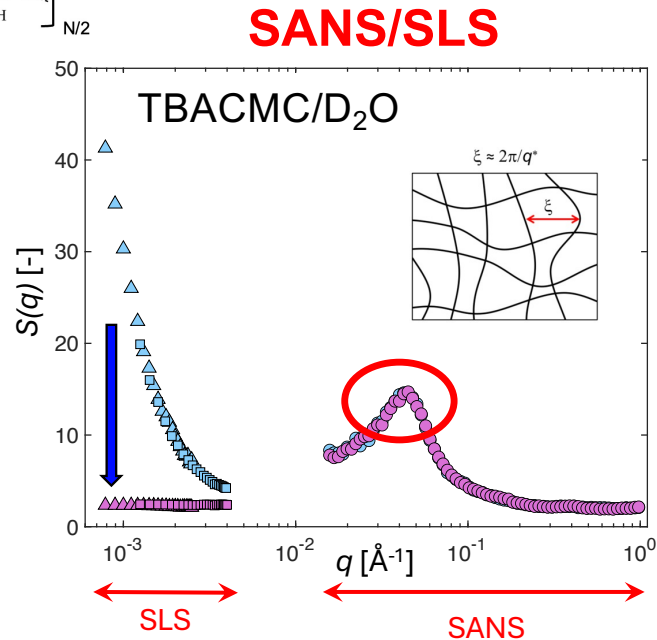
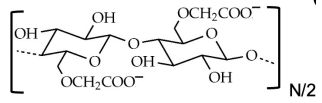
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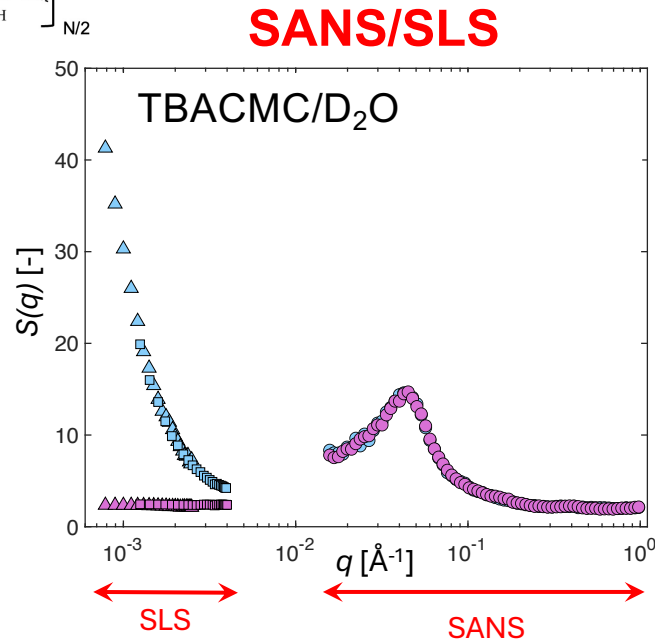
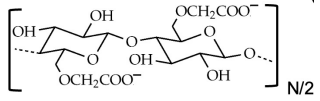
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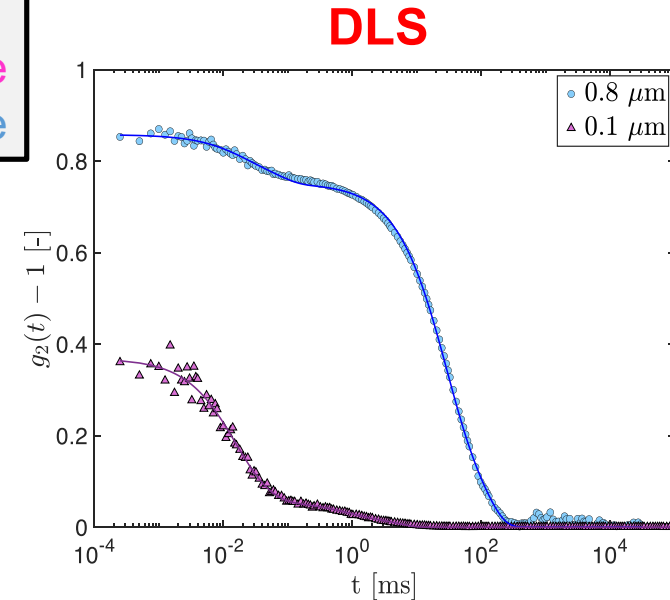
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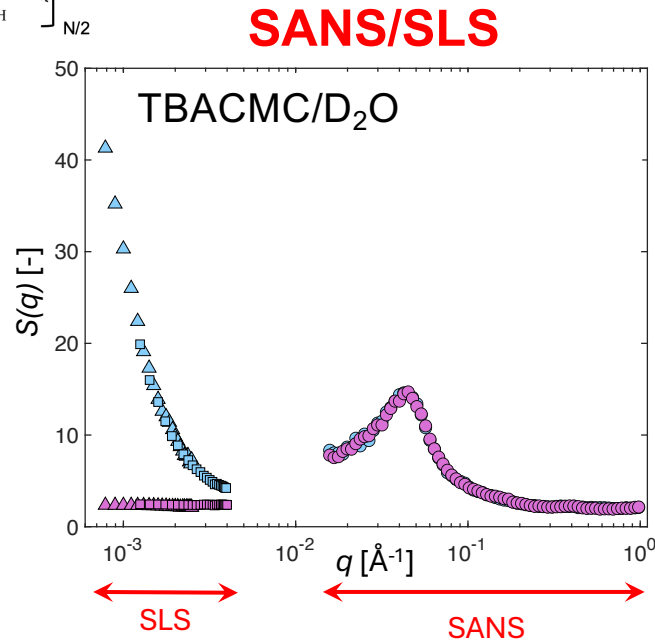
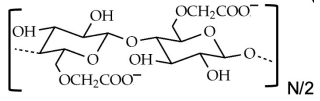
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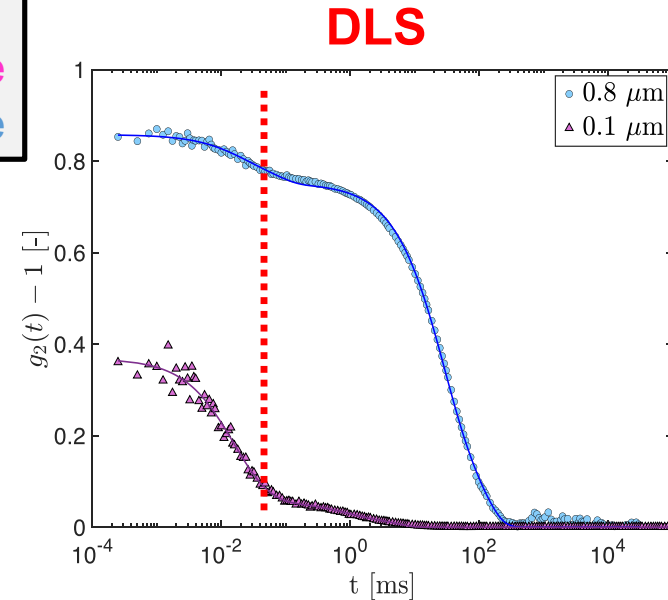
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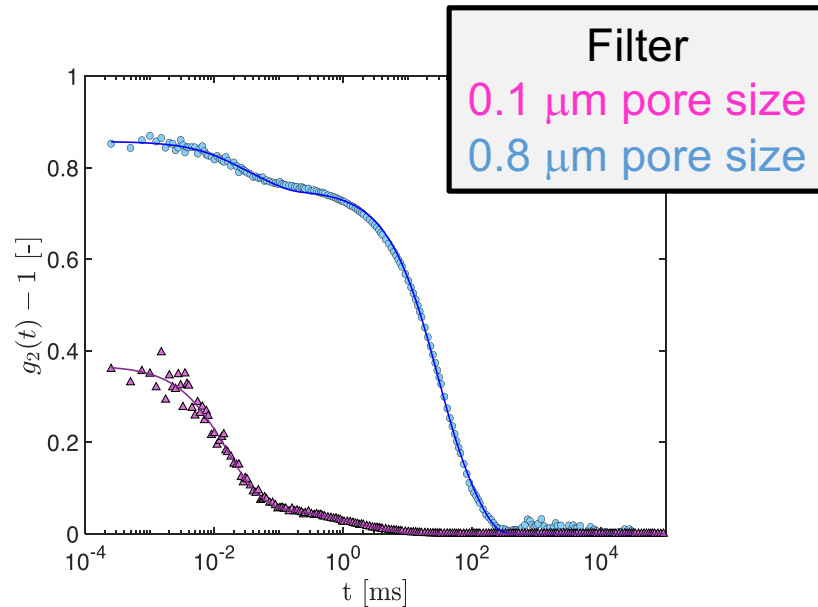
CAN WE COMPLETELY REMOVE THE LOW-Q UPTURN?

Fit autocorrelation function to bimodal decay:

$$g_1(\tau, q) = A_1(q)e^{-\Gamma_1(q)\tau} \left(1 + \frac{\mu_{2,1}t^2}{2}\right) + A_2(q)e^{-\Gamma_2(q)\tau} \left(1 + \frac{\mu_{2,2}t^2}{2}\right)$$

The relative amplitude of the fast decay in DLS is used to isolate the structure factor without the slow mode:

$$S(q)_{fast} = \frac{A_1(q)}{A_1(q) + A_2(q)} S(q)$$



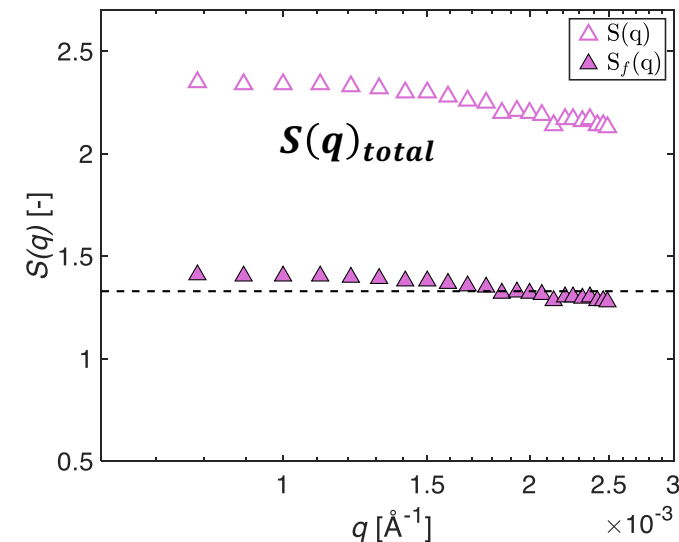
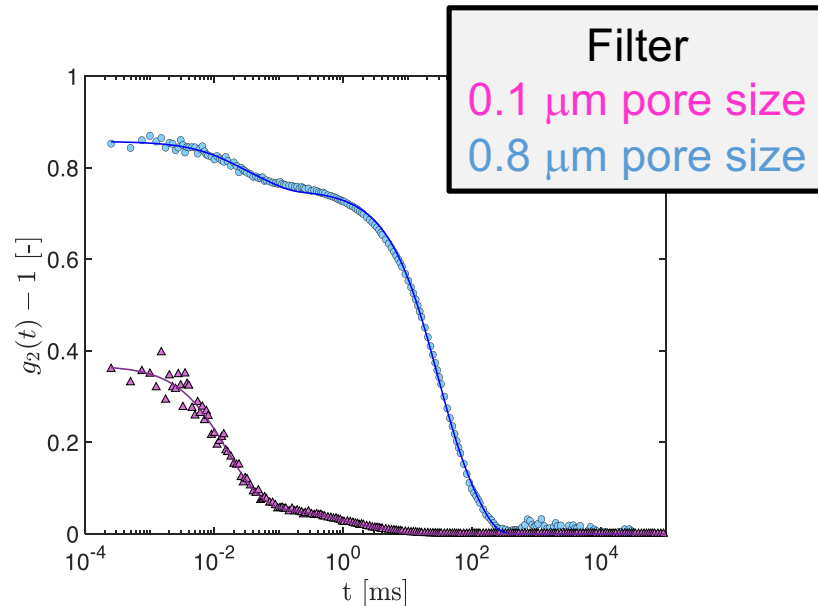
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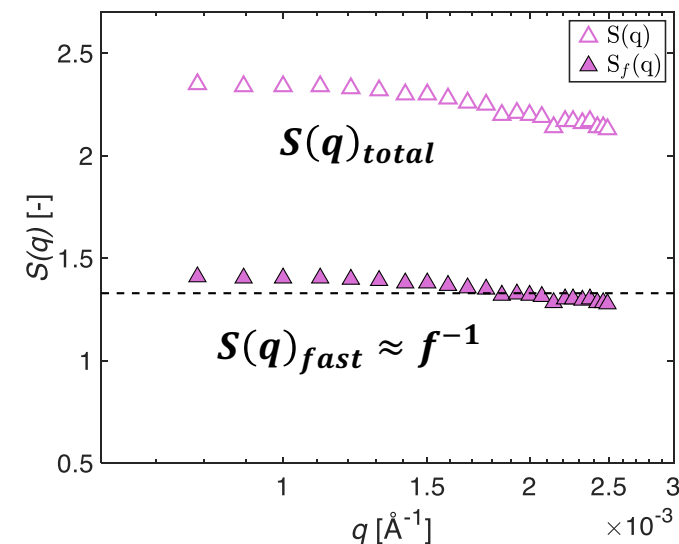
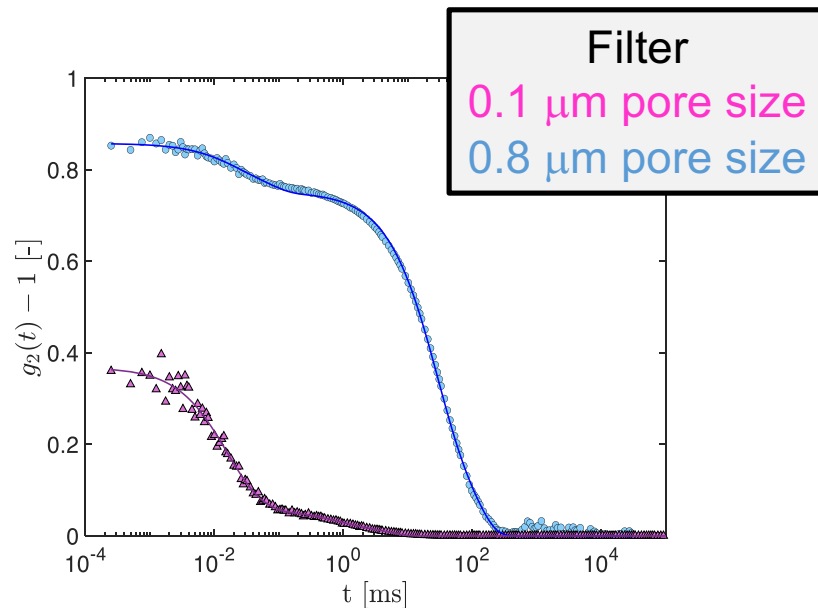
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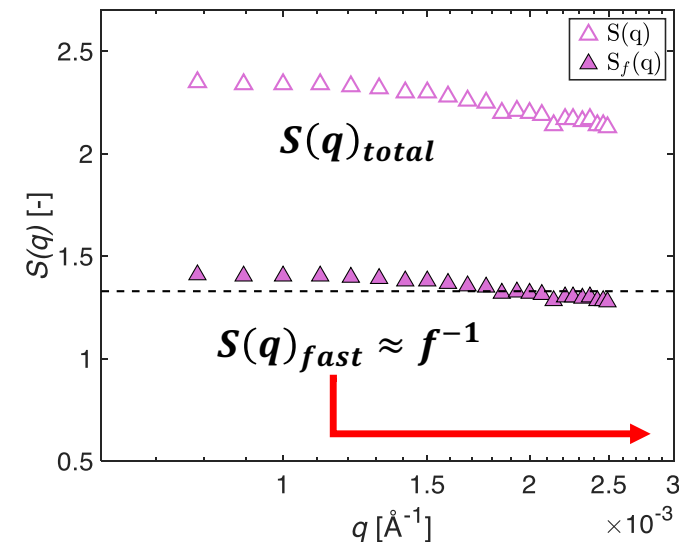
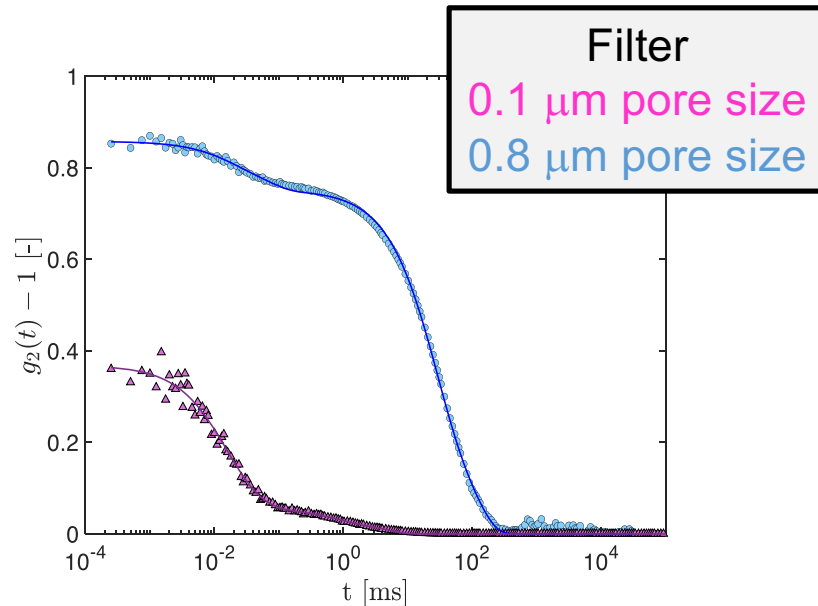
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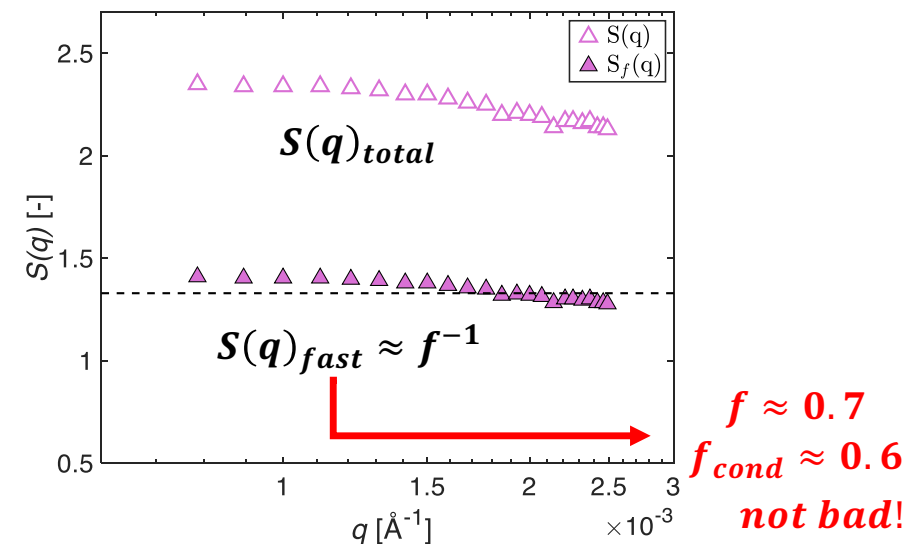
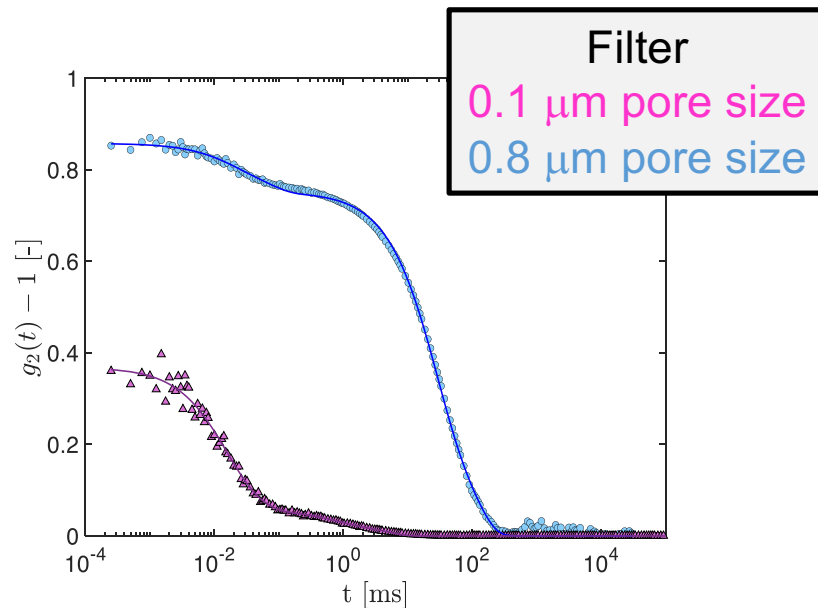
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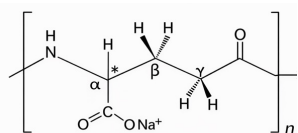
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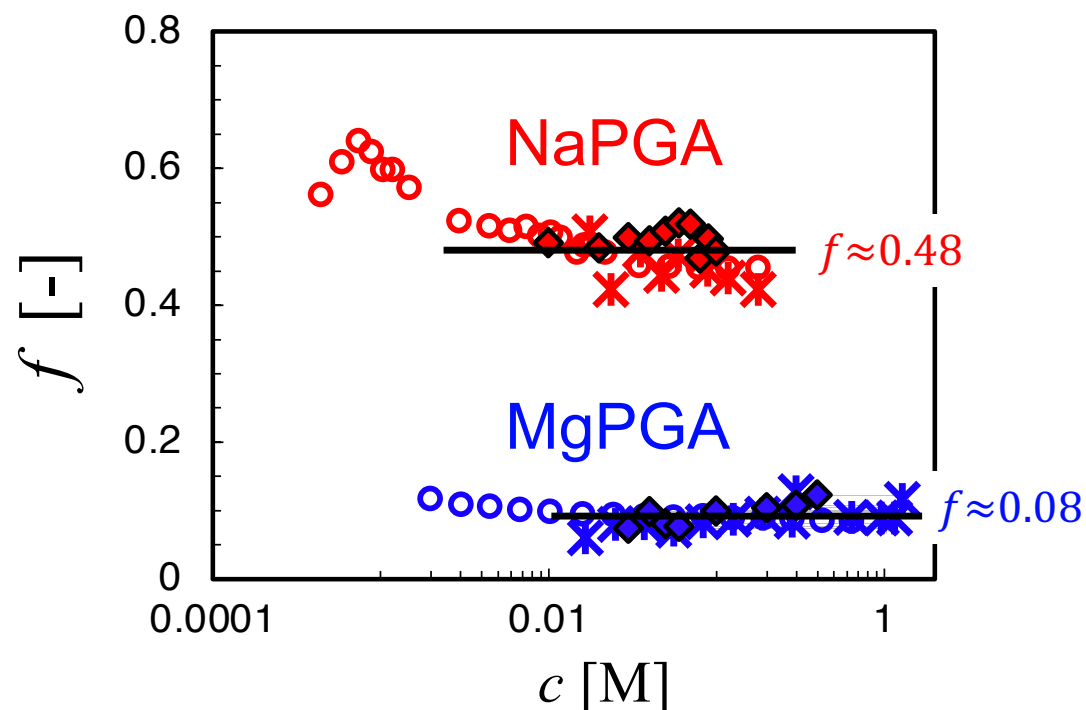
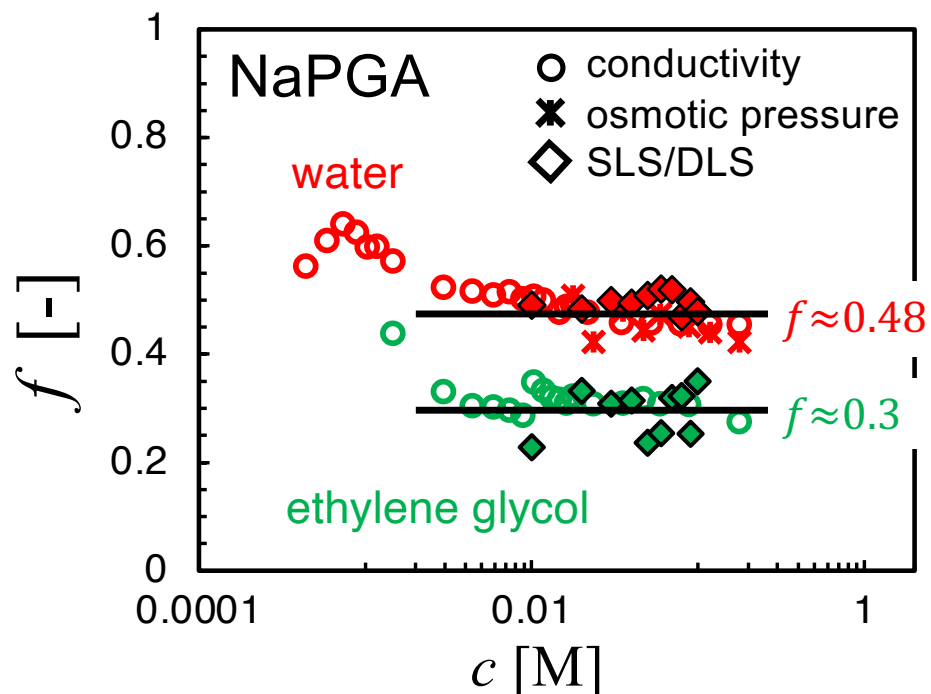
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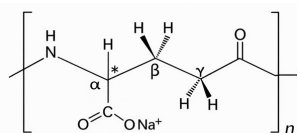




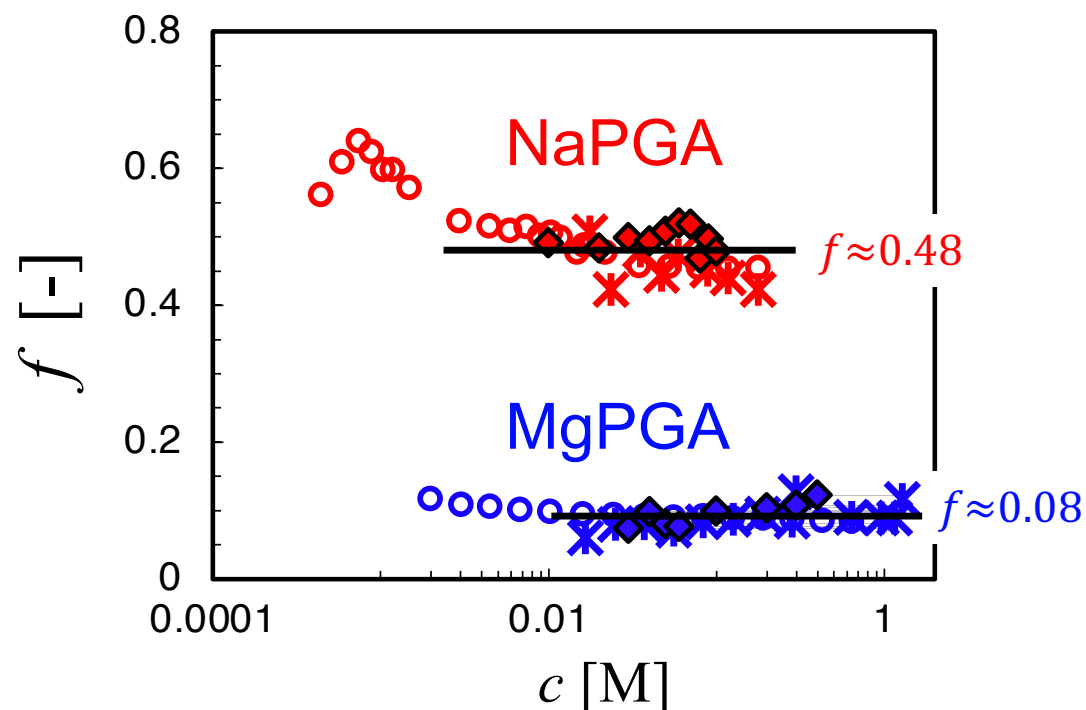
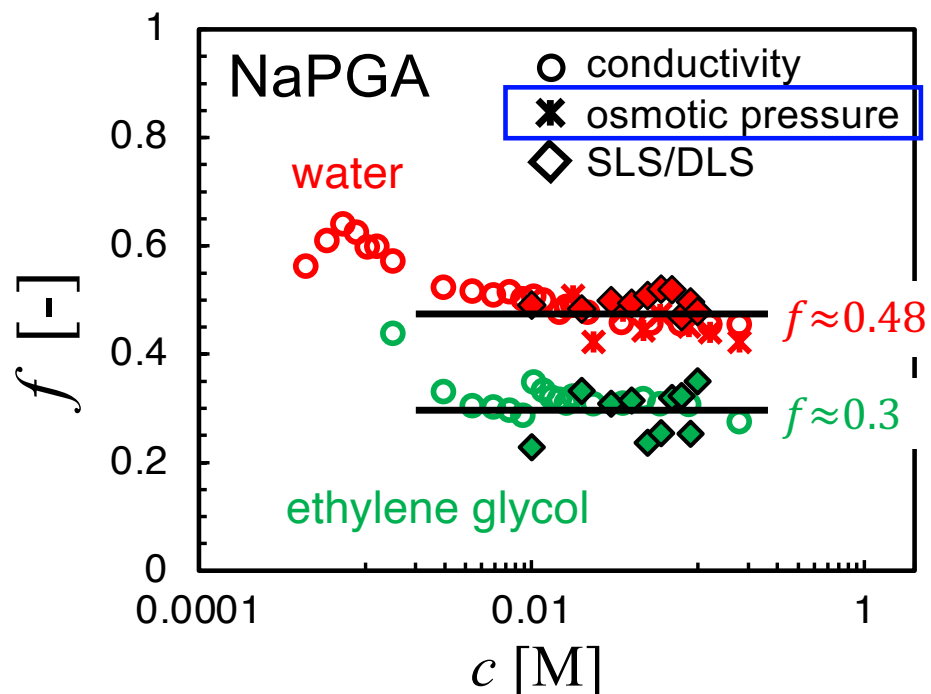
SLS/DLS METHOD: POLYGLUTAMIC ACID



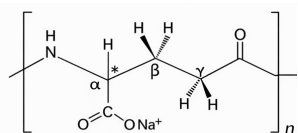
Osmotic pressure measured by SLS after removing the slow mode contribution with DLS splitting gives the correct osmotic pressure



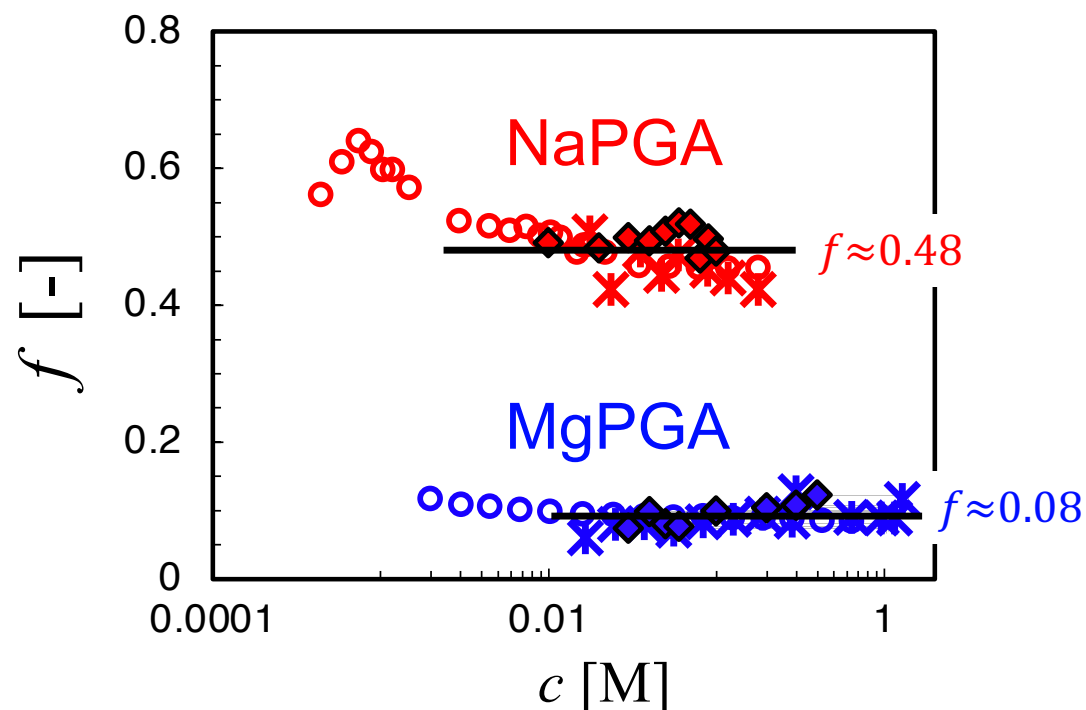
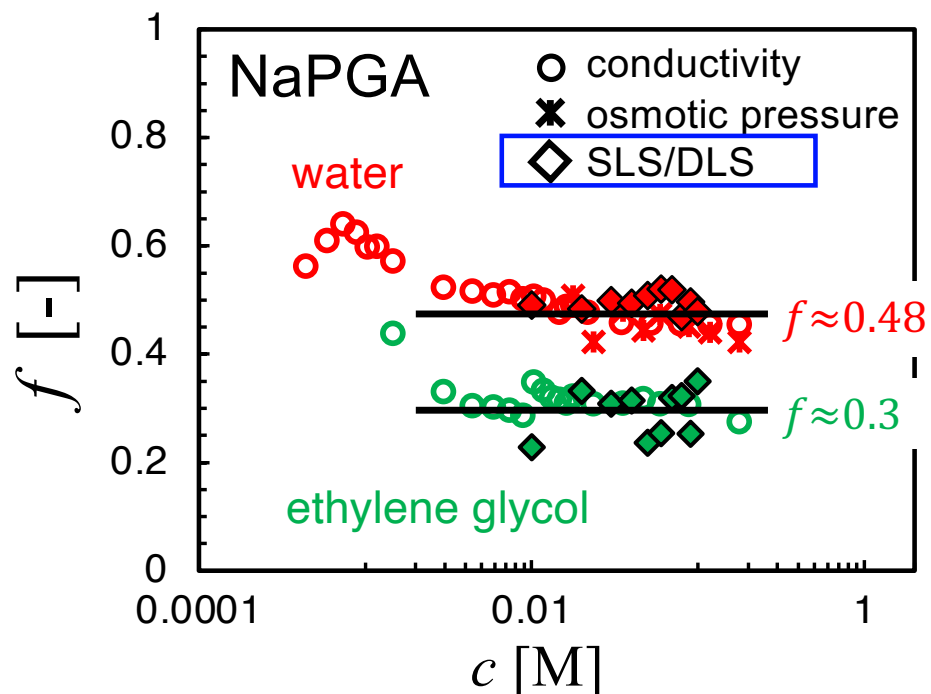
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Osmotic pressure measured by SLS after removing the slow mode contribution with DLS splitting gives the correct osmotic pressure



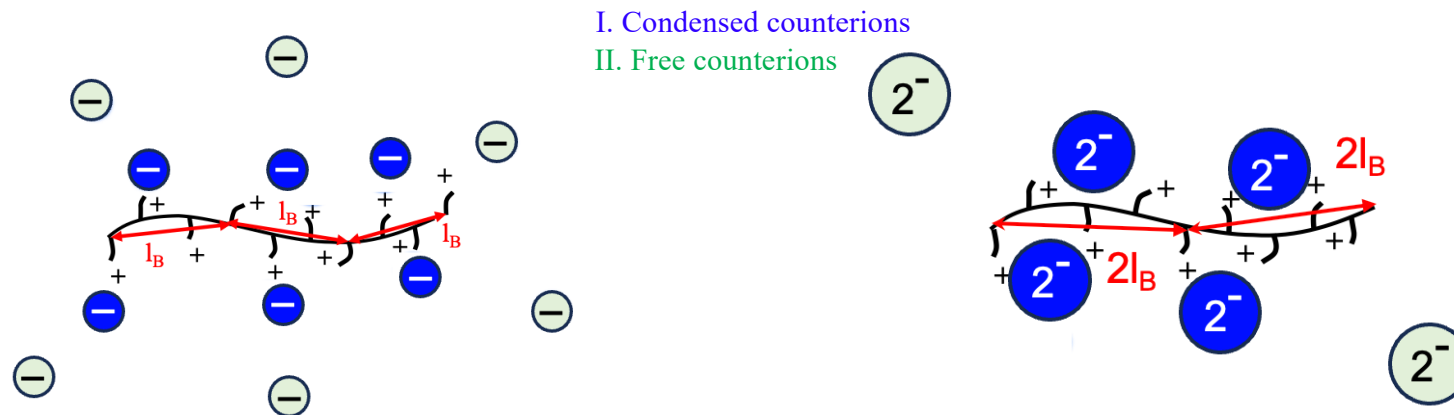
SLS/DLS METHOD: POLYGLUTAMIC ACID



Osmotic pressure measured by SLS after removing the slow mode contribution with DLS splitting gives the correct osmotic pressure

Oosawa-Manning theory of condensation

Counterions will condense onto the chain until the charge density decreases to a value of e/l_B .



Effective charge:

$$b > l_B \rightarrow f \approx 1$$

$$b < l_B \rightarrow f \approx \frac{b}{Zl_B}$$

b – distance between charges

Z – counterion valence

l_B – Bjerrum length

$$l_B = \frac{e^2}{4\pi k_B T \epsilon}$$

TEST OF OOSAWA-MANNING THEORY

Effective charge: $f \approx \frac{b}{Zl_B}$

b – distance between charges

Z – counterion valence

l_B – Bjerrum length



TEST OF OOSAWA-MANNING THEORY

$$f \approx \frac{b}{Zl_B}$$



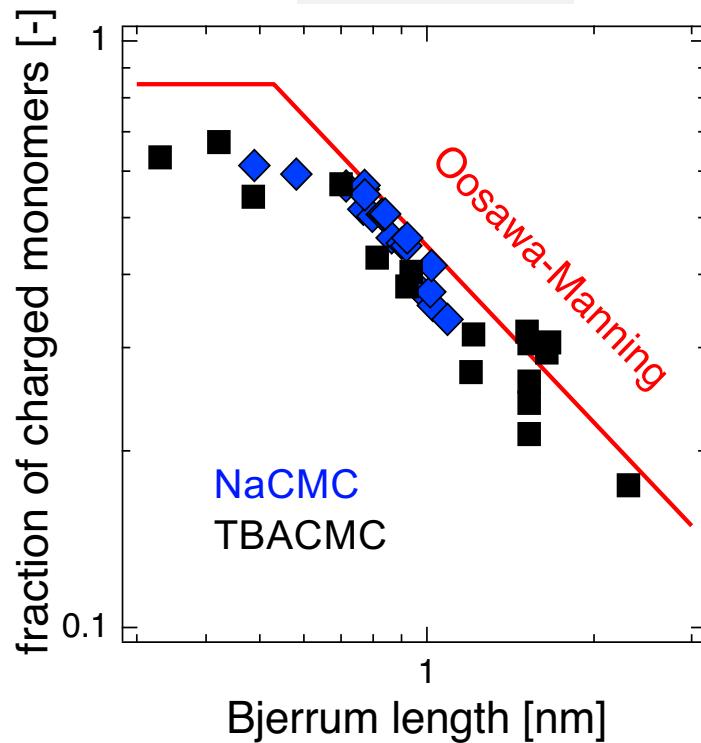
TEST OF OOSAWA-MANNING THEORY

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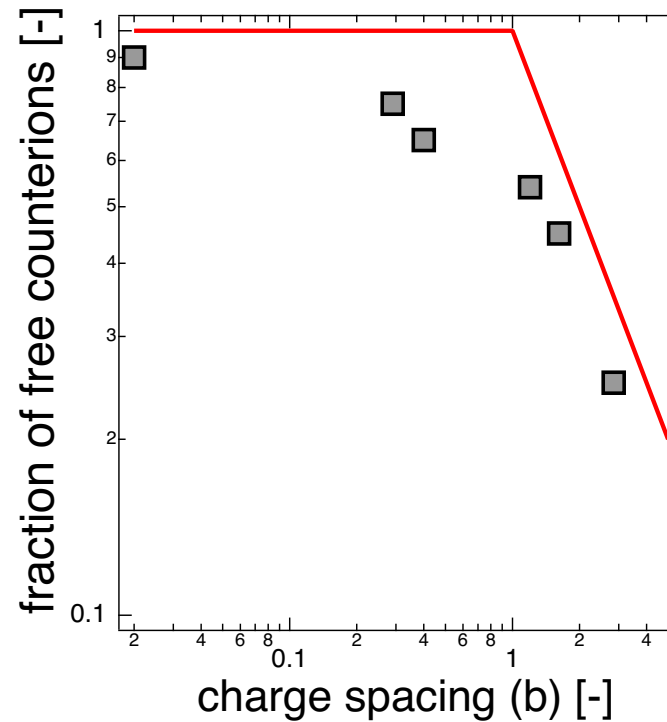
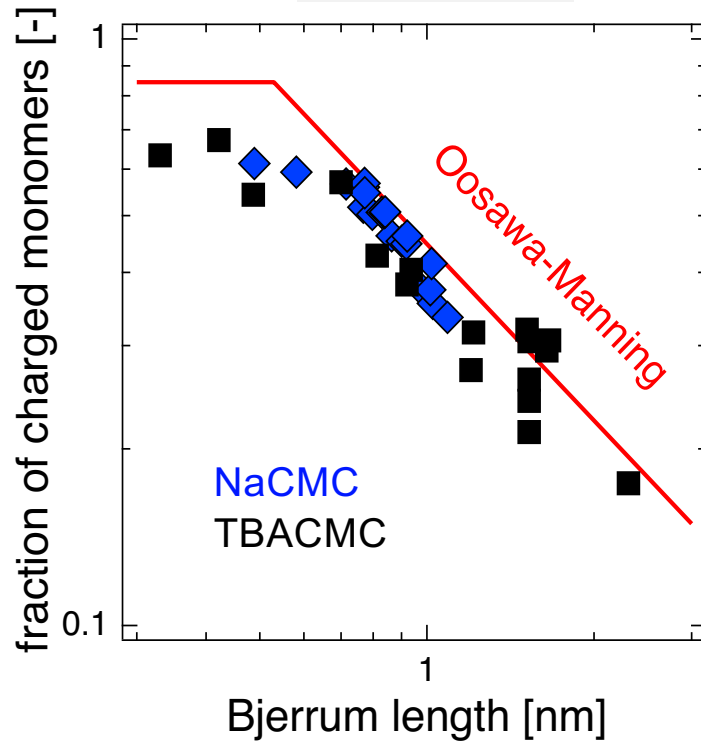


Hou C., Richtering W. & Lopez CG, Test of Oosawa–Manning condensation in a semiflexible polyelectrolyte across solvents of varying Bjerrum length *EPL* (accepted)

TEST OF OOSAWA-MANNING THEORY

$$f \approx \frac{b}{Zl_B}$$

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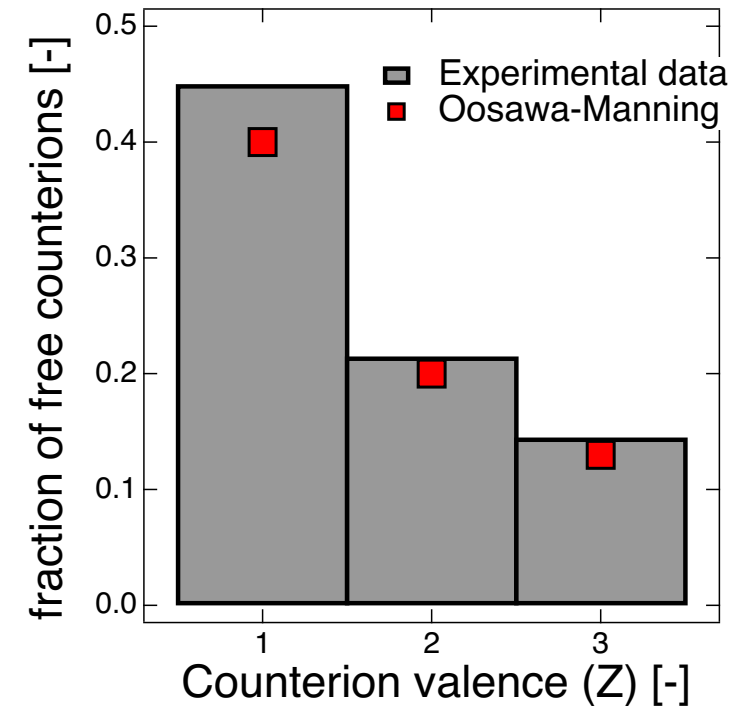
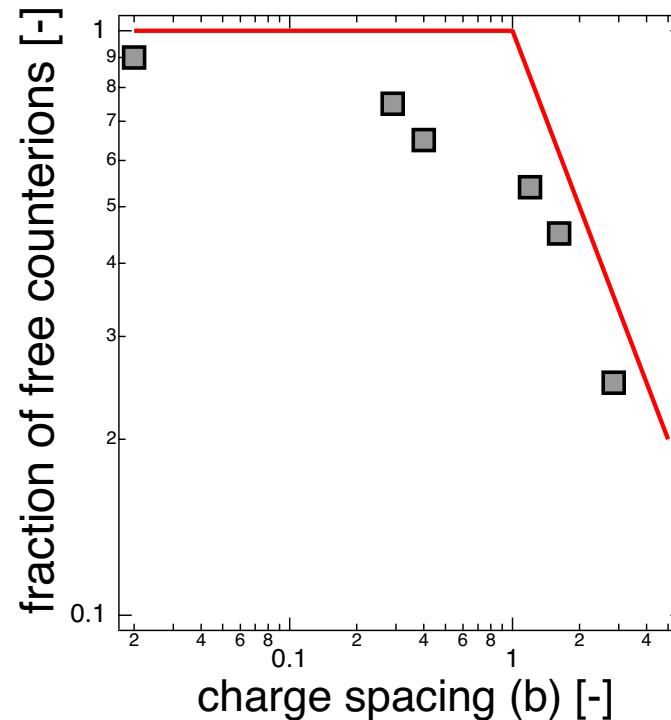
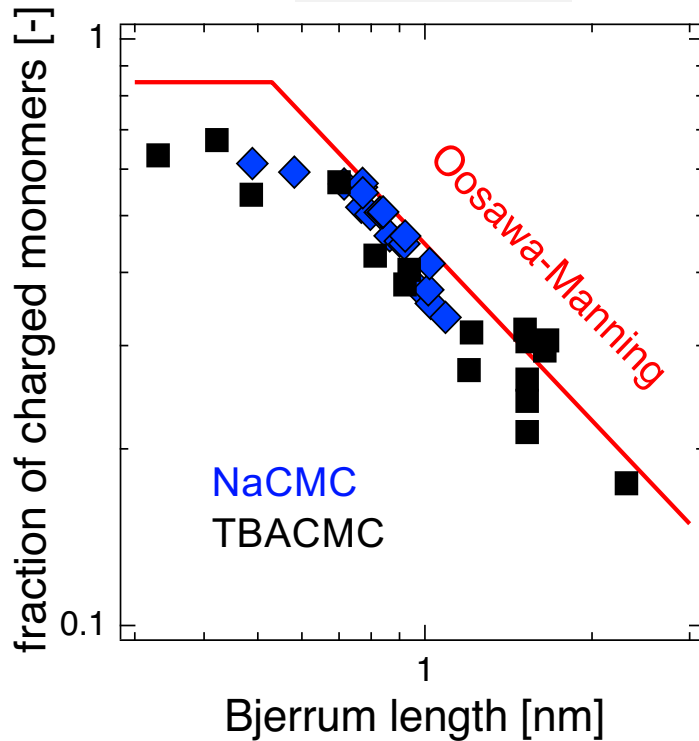


TEST OF OOSAWA-MANNING THEORY

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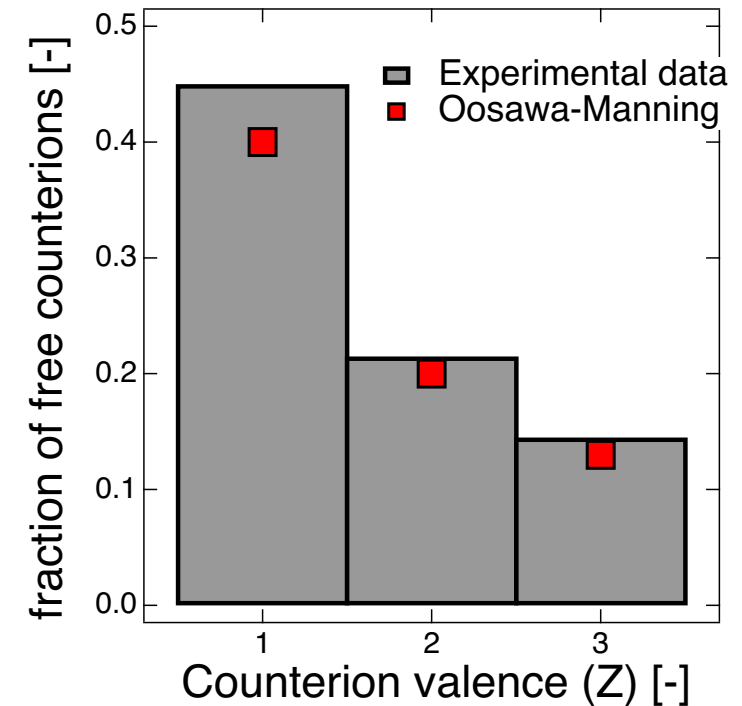
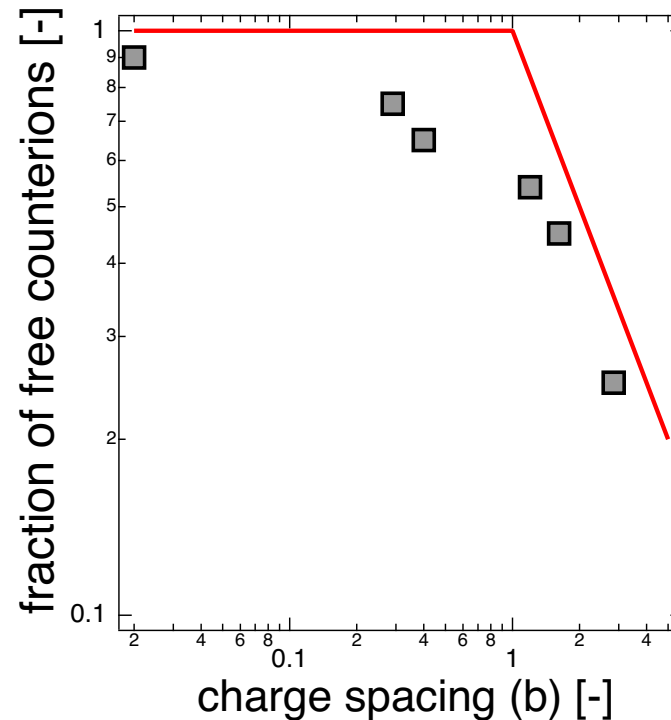
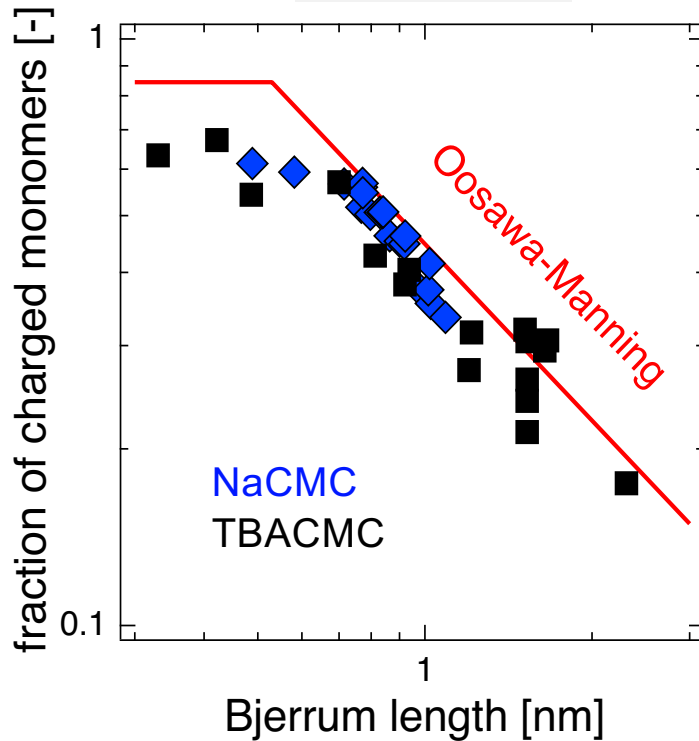


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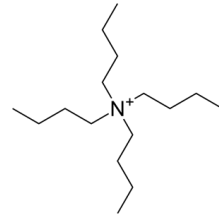
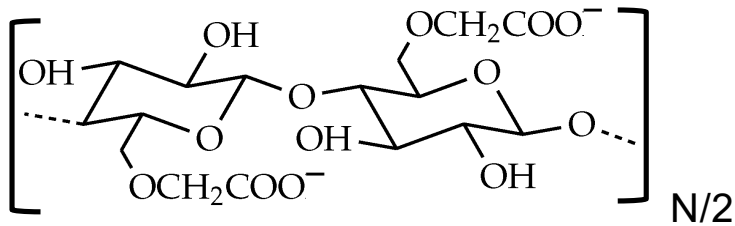


Presentation Outline

- Counterion condensation: the central problem in polyelectrolyte physics
- Light scattering as a tool to study counterion condensation
- **Beyond the two-phase model: insight from SANS and SAXS into counterion distributions**



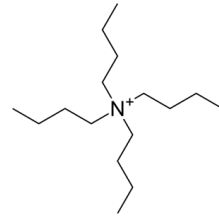
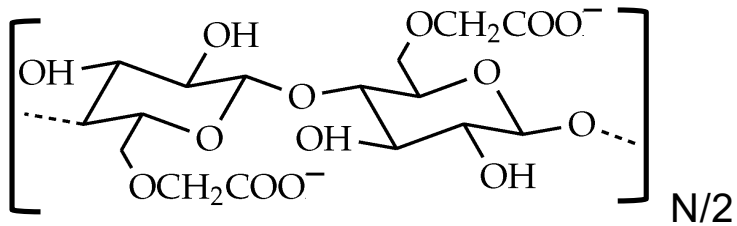
Delocalisation of hydrophobic counterions



SANS: contrast from H-rich counterions

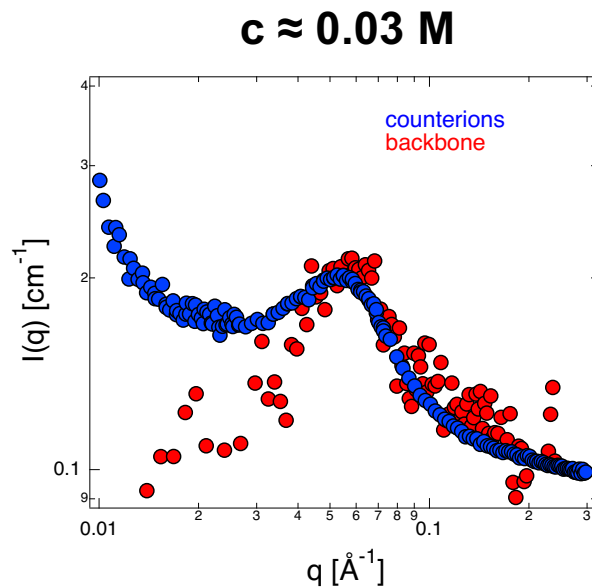
SAXS: contrast from polymer backbone

Delocalisation of hydrophobic counterions

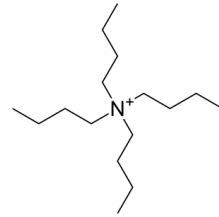
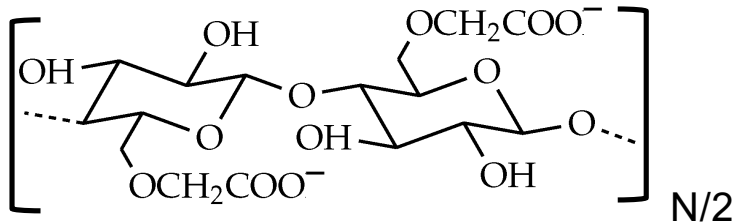


SANS: contrast from H-rich counterions

SAXS: contrast from polymer backbone

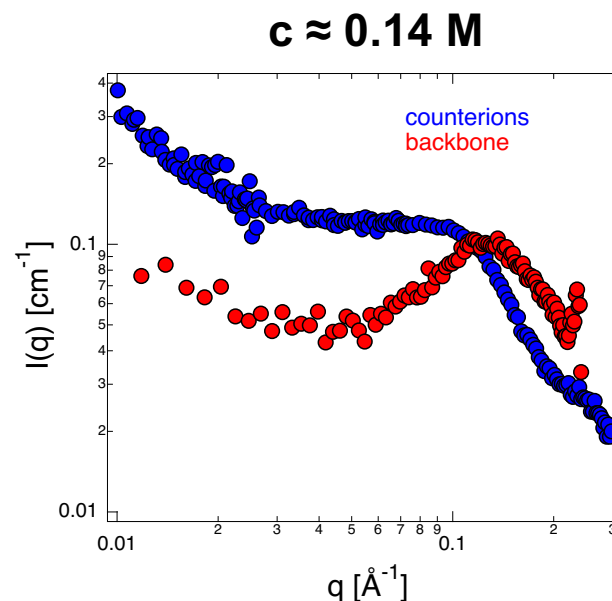
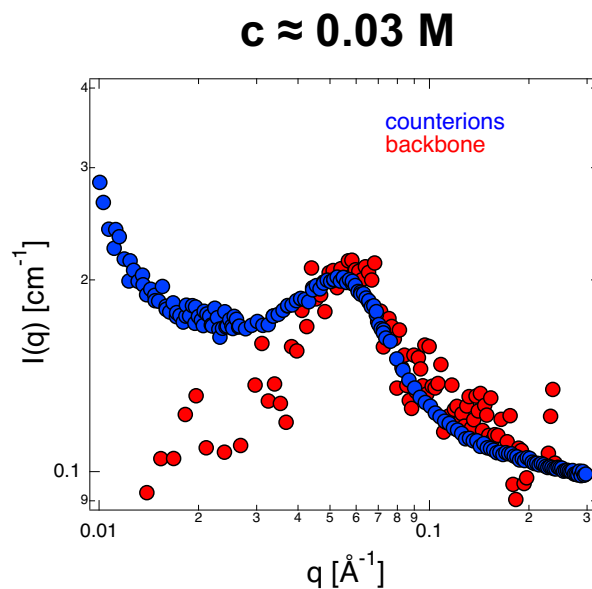


Delocalisation of hydrophobic counterions



SANS: contrast from H-rich counterions

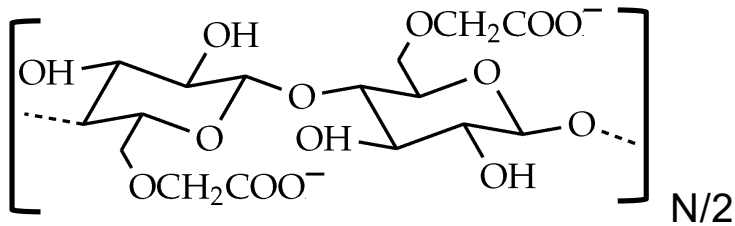
SAXS: contrast from polymer backbone



Hydrophobic counterions de-localise from the backbone at high concentrations



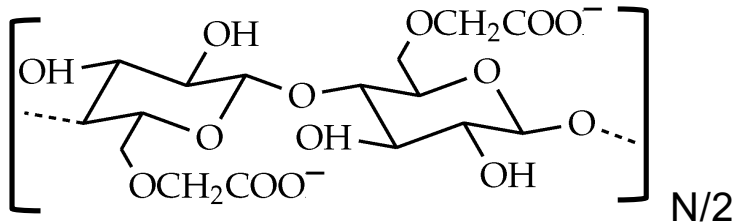
Effect not observed for hydrophilic ions



SANS: contrast from H-rich backbone

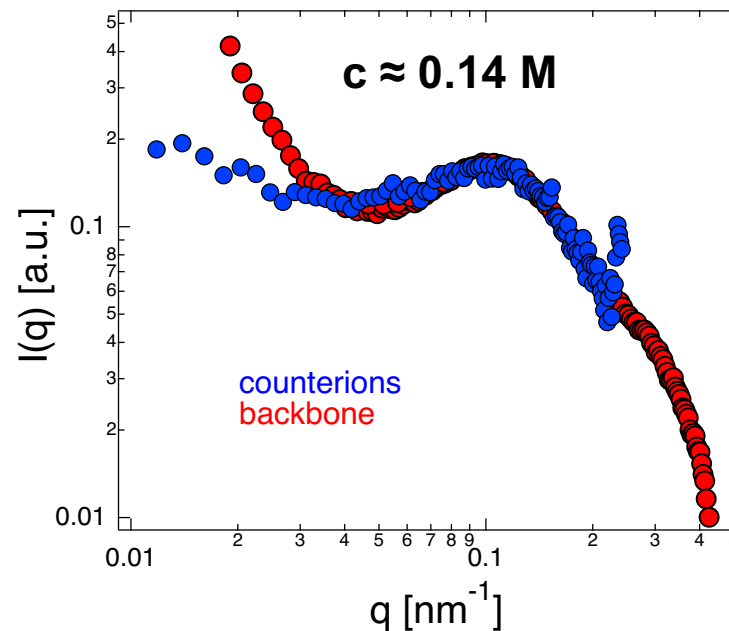
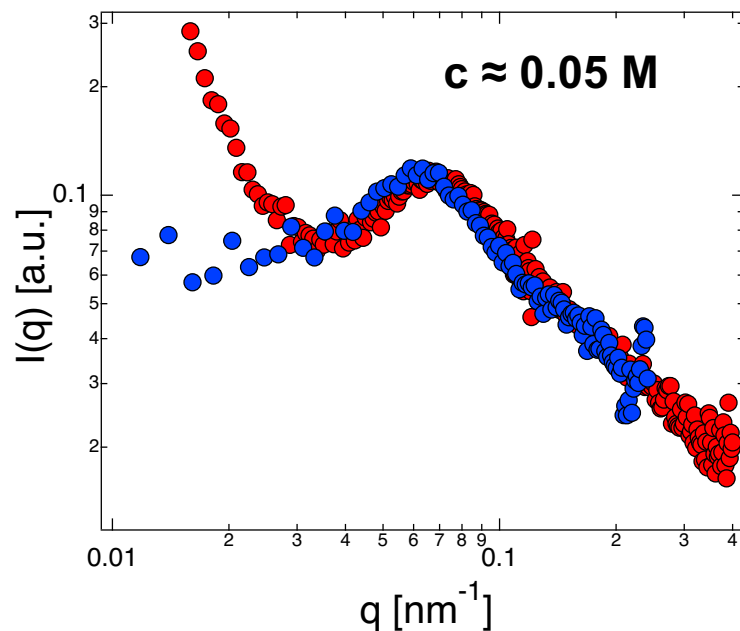
SAXS: contrast from electron-rich counterion

Effect not observed for hydrophilic ions



SANS: contrast from H-rich backbone

SAXS: contrast from electron-rich counterion

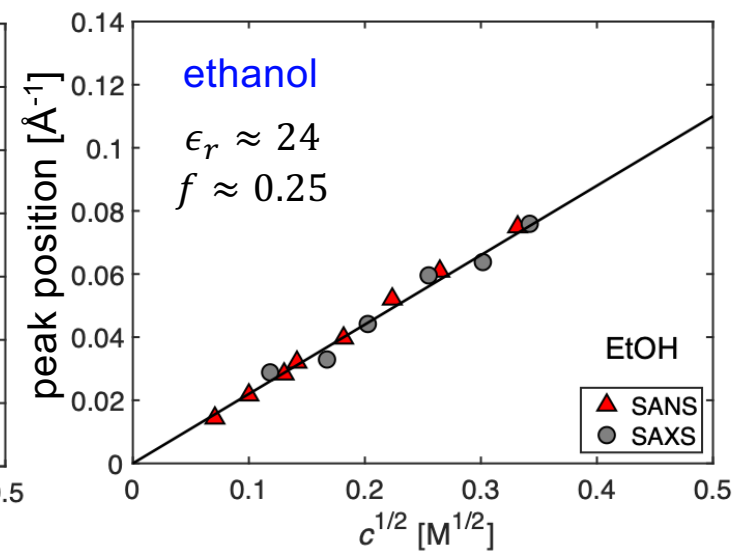
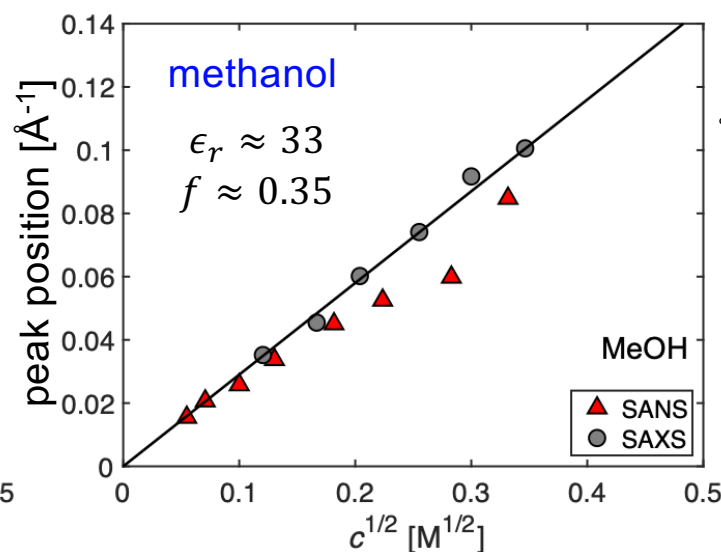
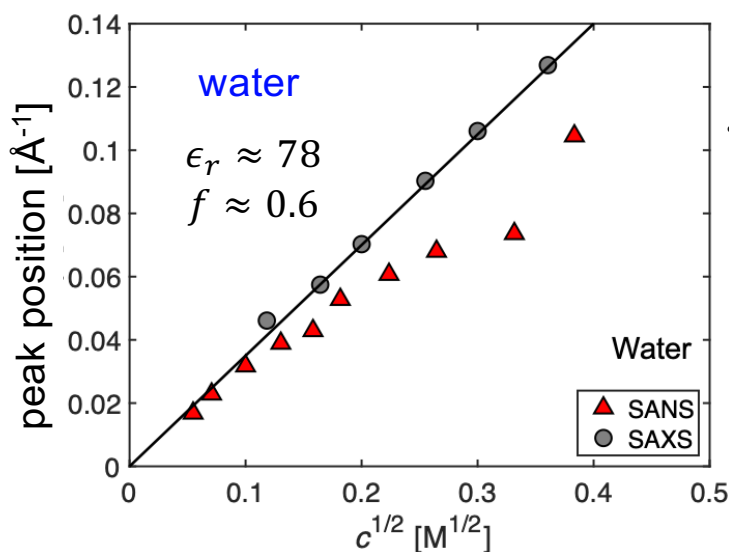


Hydrophilic counterions remain tightly correlated with the backbone at low and high concentrations



POLYMER-COUNTERION DECOUPLING

Contrast from polymer backbone
Contrast from counterions



Dielectric constant decreases

Polymer & counterion fluctuations de-couple

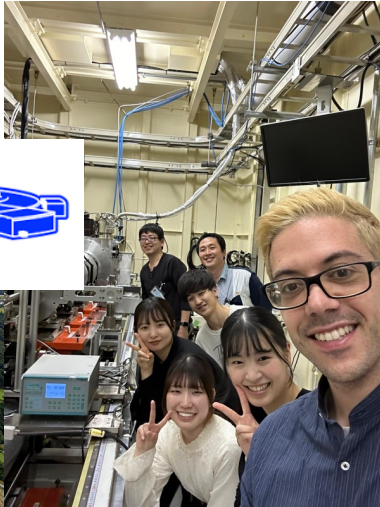
Gulati, A., Meng, L., Hou, C., Watanabe, T. and Lopez, C.G., 2026. Solution structure and counterion condensation of carboxymethyl cellulose in organic solvents. *ACS Applied Polymer Materials*. 8, 11, 8183–8197

CONCLUSIONS

- Combined static and dynamic **light scattering offers a way to measure the osmotic pressure of polyelectrolytes.**
- Oosawa-Manning correctly predicts the fraction of free counterions for semiflexible polyelectrolytes.
- SANS and SAXS can be used to probe the local concentration fluctuations of counterions and polyion backbone. **Hydrophobic counterions de-couple** from the backbone at high concentrations while hydrophilic ones do not.

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(Didcot, UK)**



Ralph Colby
(PSU)



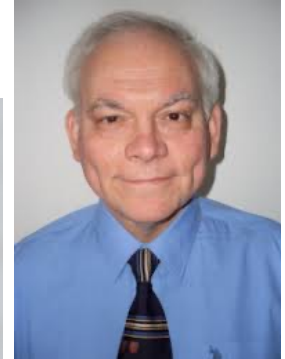
W. Richtering
(RWTH Aachen)



Jack Douglas



Ferenc Horkay



Preprints:



Japan Research Reactor-3

Effect of dielectric constant

